Mathematical Modelling of Heating and Evaporation of a Spheroidal Droplet

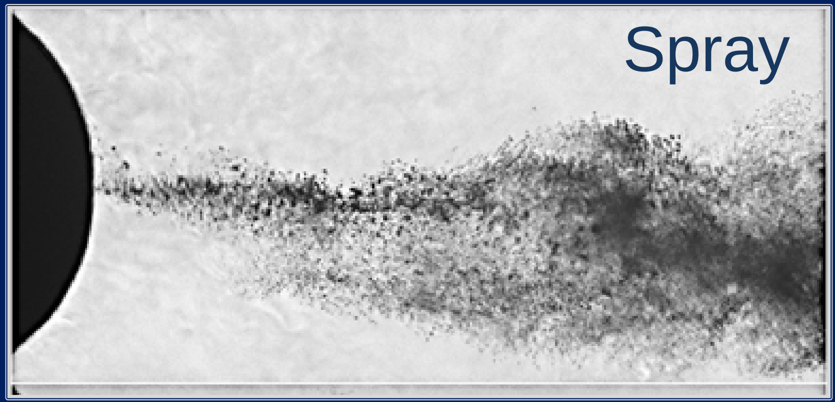
Vladimir Zubkov

S. Tonini, G.E. Cossali, C. Crua, M. Heikal, S. Sazhin

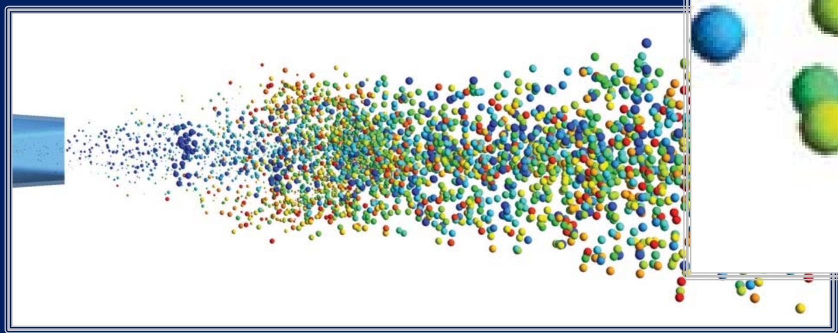
EPSRC
Engineering and Physical Sciences Research Council

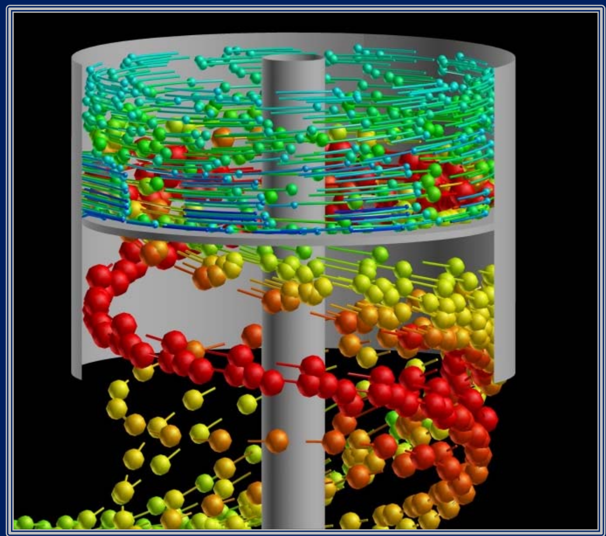
University of Brighton
Advanced Engineering Centre

Problem Statement

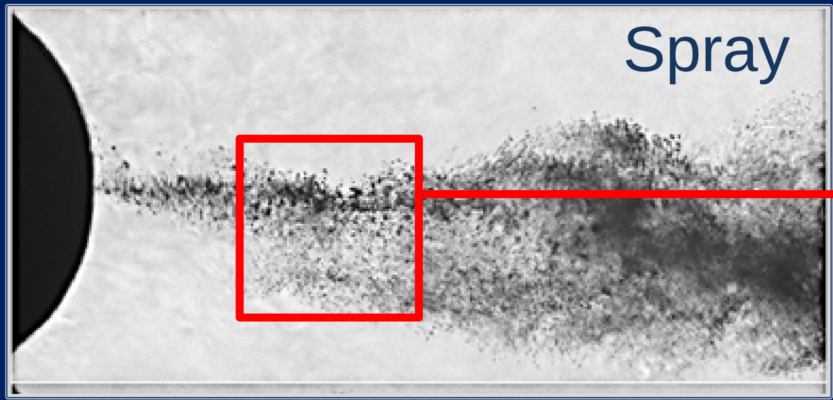


Current mathematical models of sprays use spherical approximation for droplets.

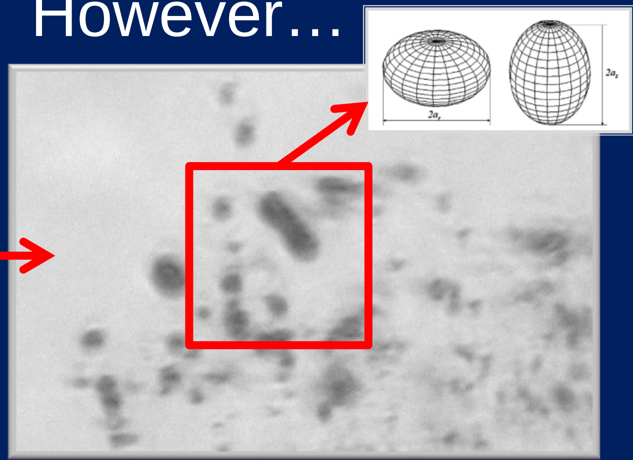




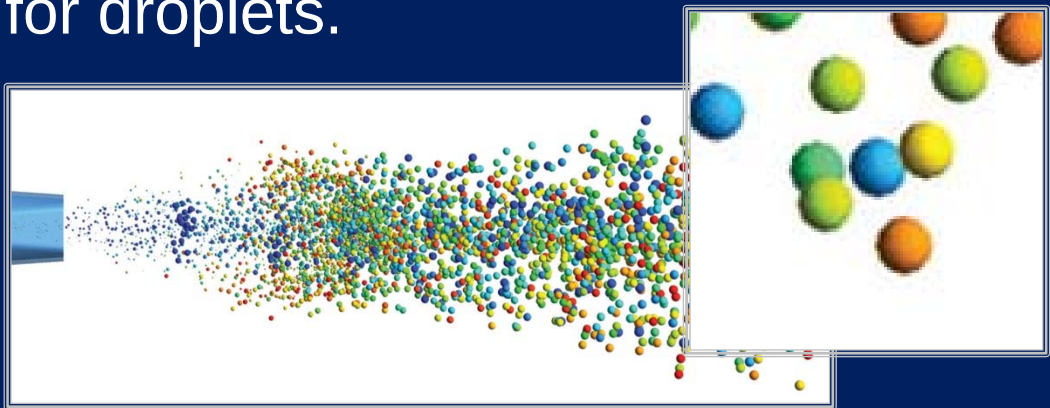
Problem Statement

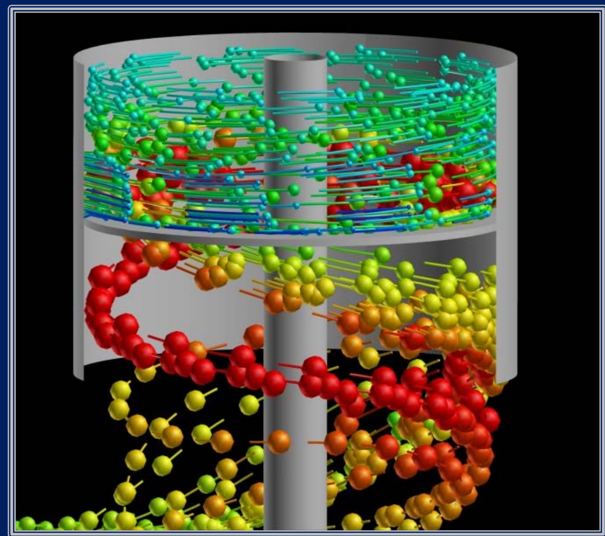


However…



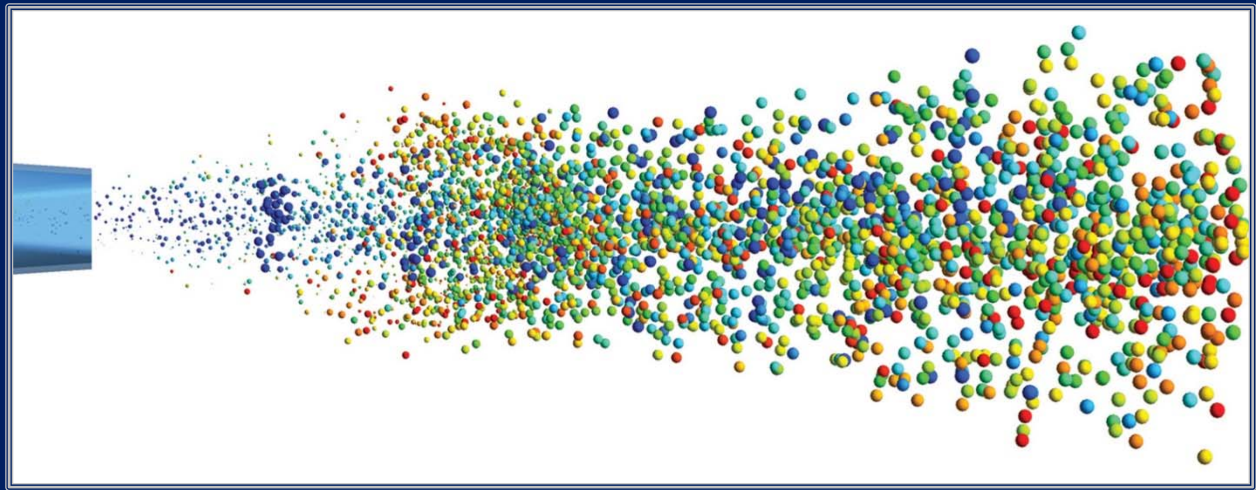
Current mathematical models of sprays use spherical approximation for droplets.



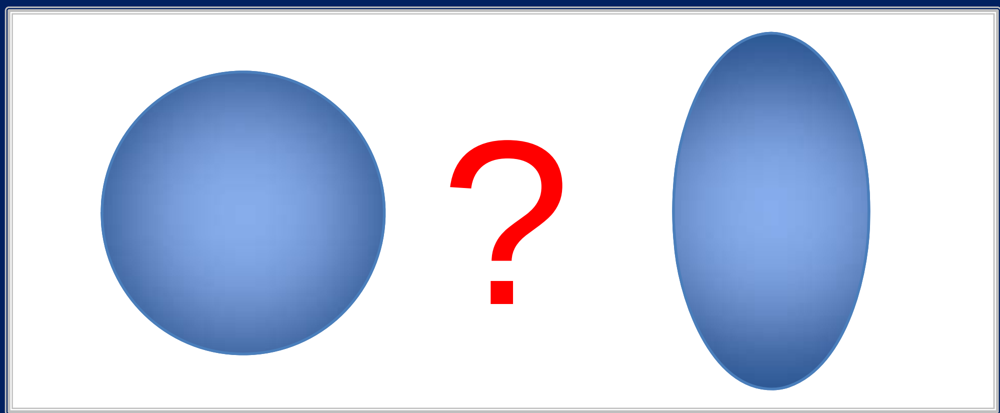


Problem Statement

Can the spherical approximation of droplets be used?

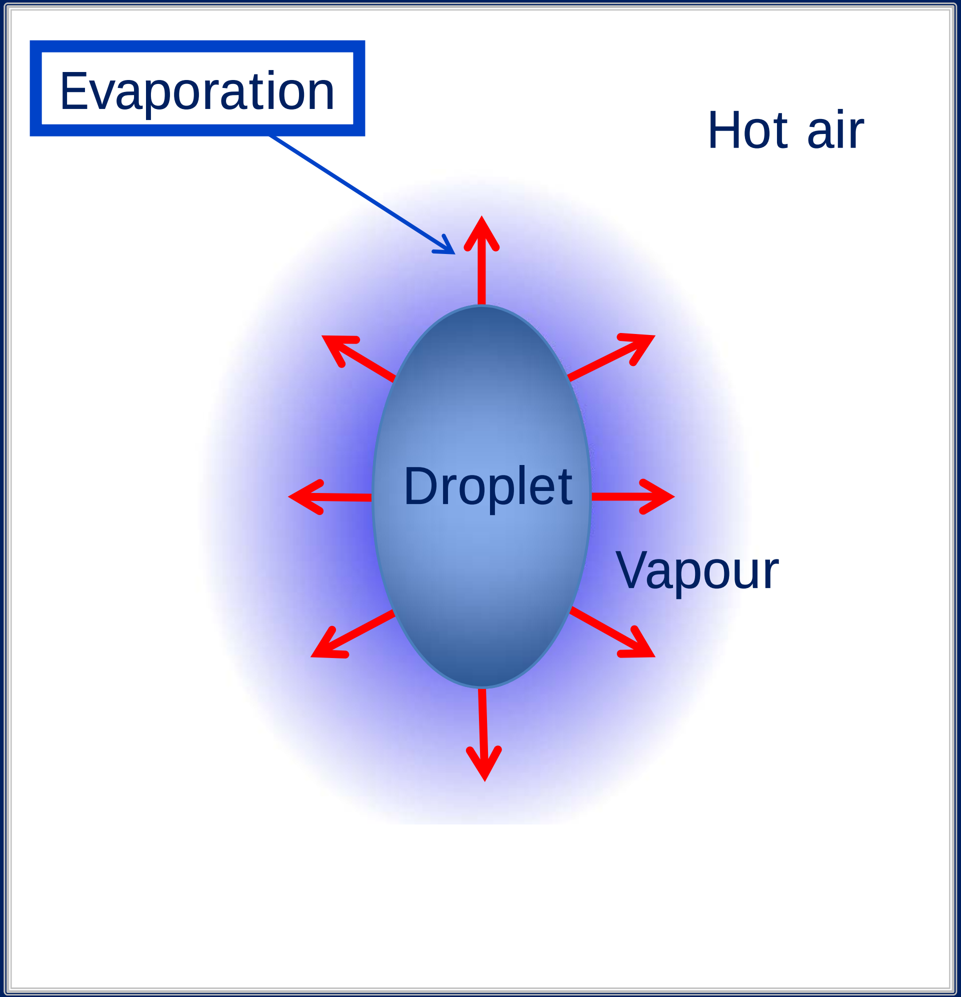


We will consider heating and evaporation of a single non-spherical droplet and compare it with a spherical one.



Problem Statement

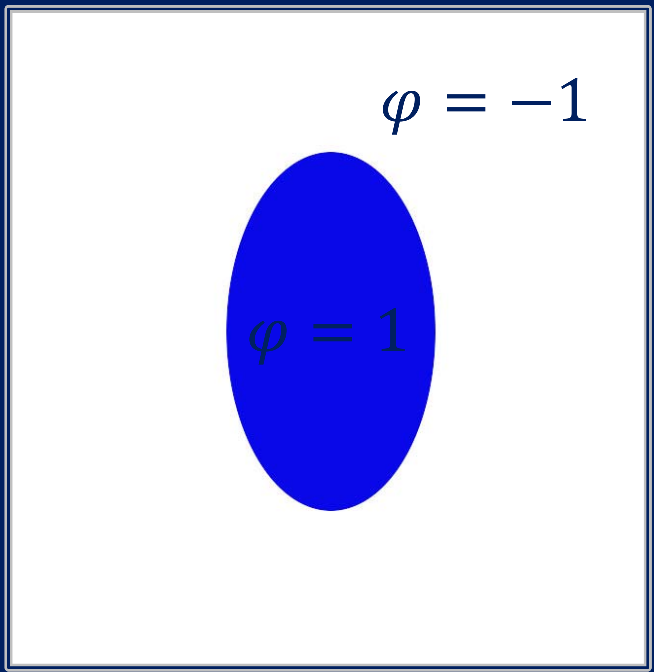
Heating and evaporation of a single non-spherical droplet.



Mathematical Modelling

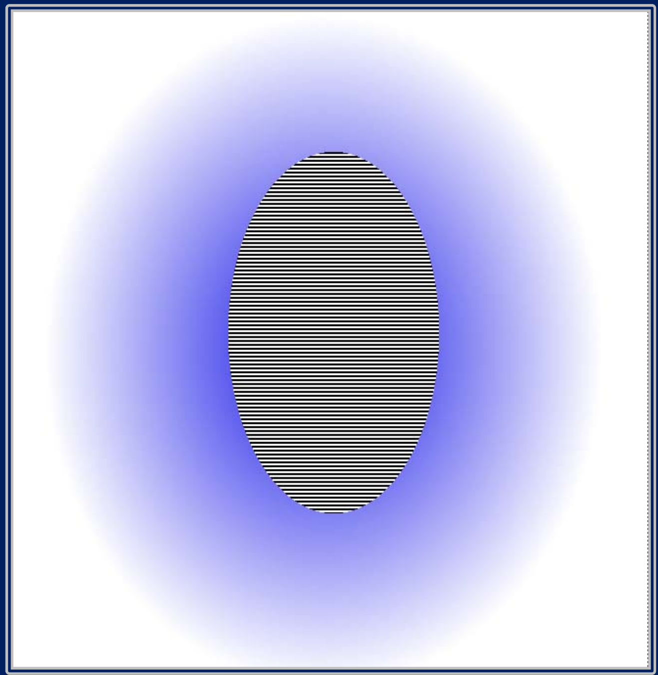
Droplet dynamics:

Phase-field method – liquid and gas



Vapour dynamics:

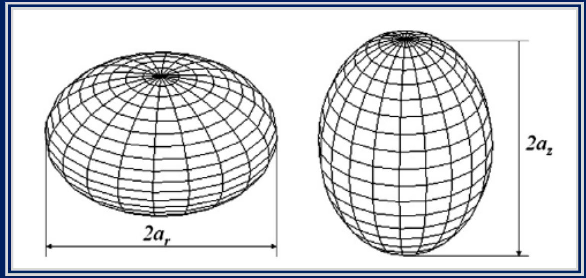
Mixture Model of two phases – vapour and air



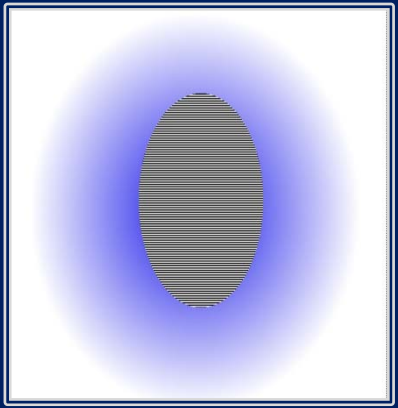
It is hard to couple these two models: we have three phases

Model simplifications

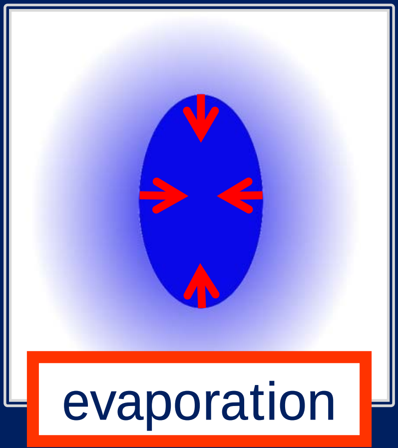
1. Approximate droplets by spheroids: 3D => 2D



2. Find temperature and vapour distribution for a steady state assuming that droplet shape is fixed and droplet temperature at the surface is uniform.

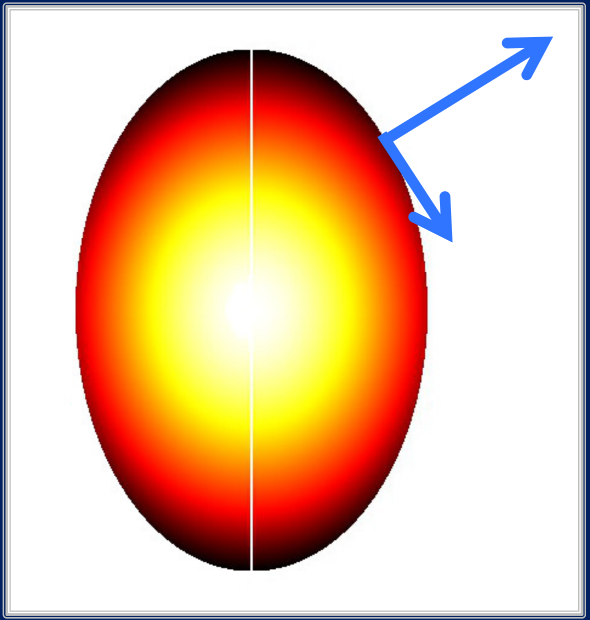


3. Use exact solution from for vapour in the boundary condition to calculate the reassertion of droplet surface and distribution of T inside the droplet.



Additional simplifications

We consider only slightly deformed droplets to guaranty that temperature gradient in radial direction is much larger than in the tangential direction.

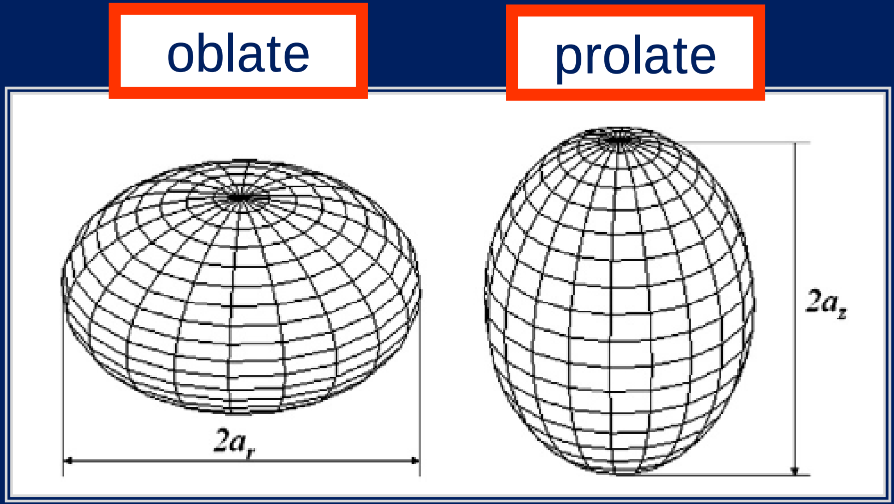


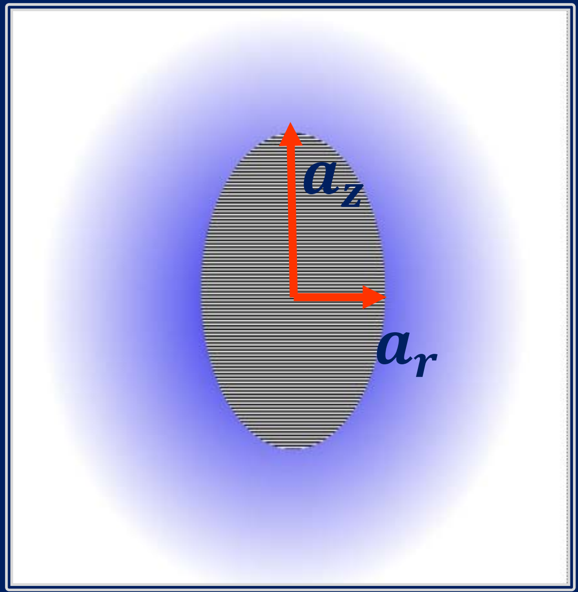
We ignore:

- Droplet swelling (changes of the droplet volume due to changes in temperature);
- Surface tension and resulting oscillation of the droplet;

Previous models

S. Tonini, G.E. Cossali (2014) proposed a mathematical model for non-spherical droplets:





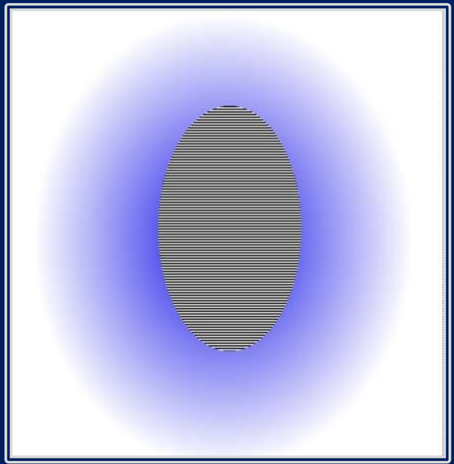
Approximations:

$$\varepsilon = a_z / a_r$$

- Problem is solved outside the droplet
- Droplet shape is fixed
- Temperature is constant on the droplet surface

Tonini & Cossali model

The steady-state evaporation of a spheroidal drop can be analysed through the solution to the species balance equations:



species (air or vapour)

mass diffusivity

$$\nabla_j\left(\rho U_j \varphi^{\alpha} - \rho D_v \nabla_j \varphi^{\alpha}\right) = 0$$

$$\varphi^{\alpha} = \rho^{\alpha}/\rho$$

mass fraction

total density

Air and vapour can move due to convection and diffusion

Tonini & Cossali Results

In the spheroidal coordinate system the problem was solved analytically:

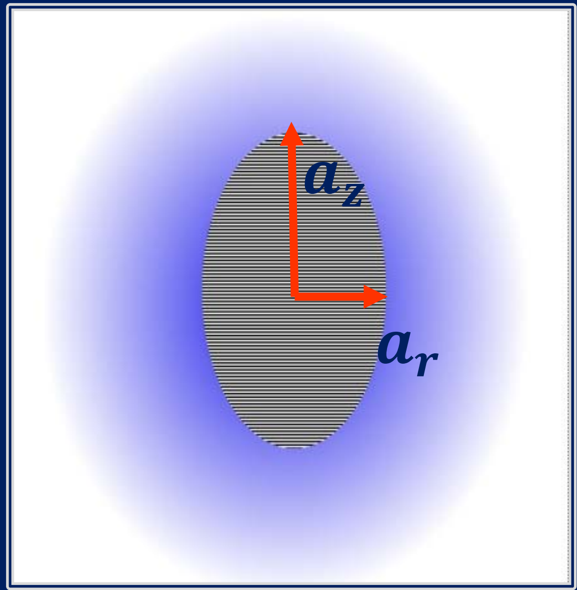
Evaporation rate of the droplet:

$$m_{ev} = C(\varepsilon) ln \frac{1}{1-\varphi_s^v}$$

evaporation factor

mass fraction

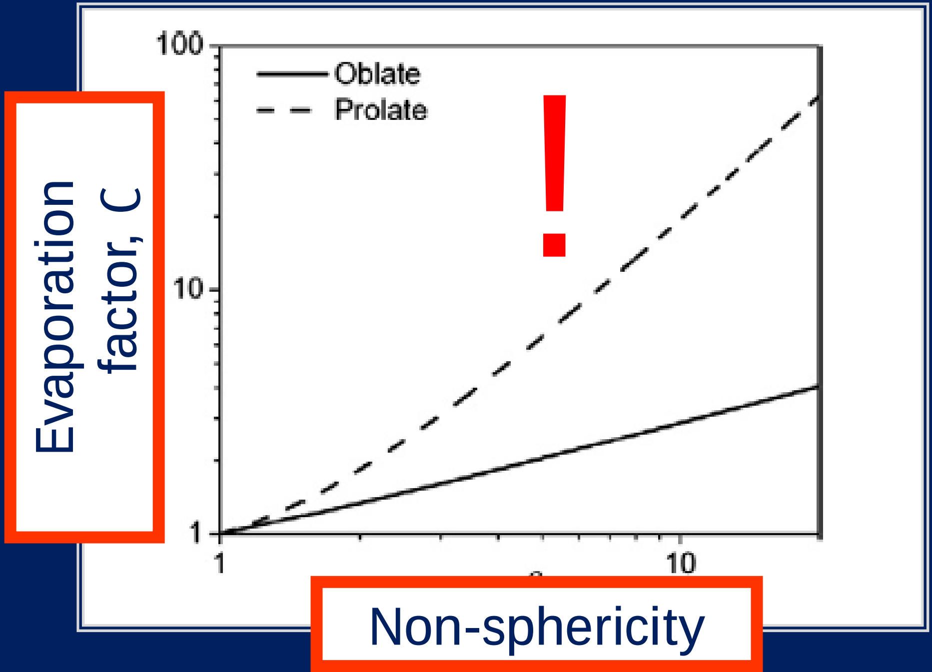
$C(\varepsilon) = 1, \quad \text{at} \quad \varepsilon = 1$

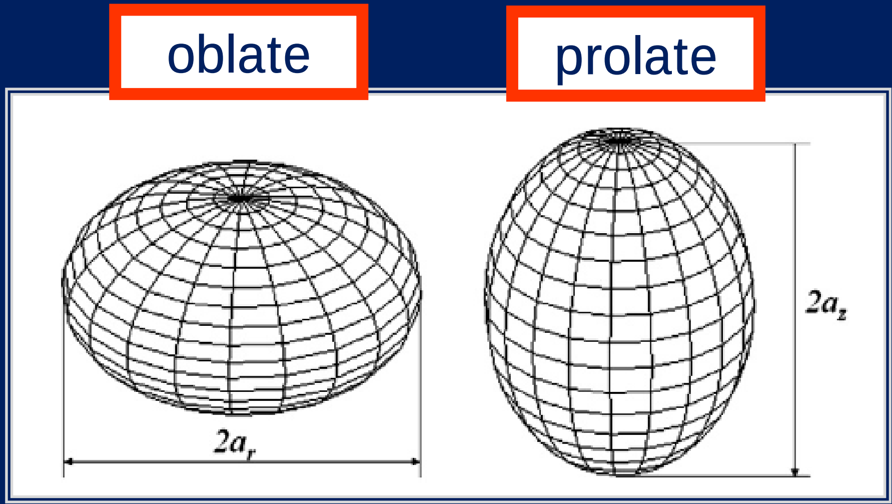


$\varepsilon = a_z/a_r$

Tonini & Cossali Results

Non-spherical droplets evaporates faster.





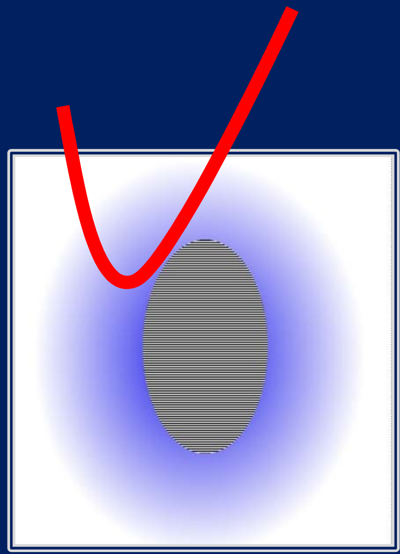
Non-physical approximations:

- Droplet shape is fixed
- Temperature is constant inside and at the surface of the droplet

Model Development

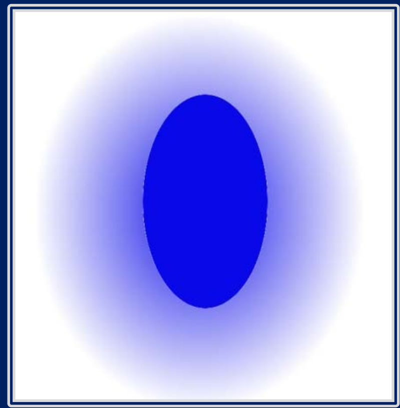
Approximations:

- S. Tonini, G.E. Cossali (2014) mathematical model for non-spherical droplet for the gas phase outside the droplet



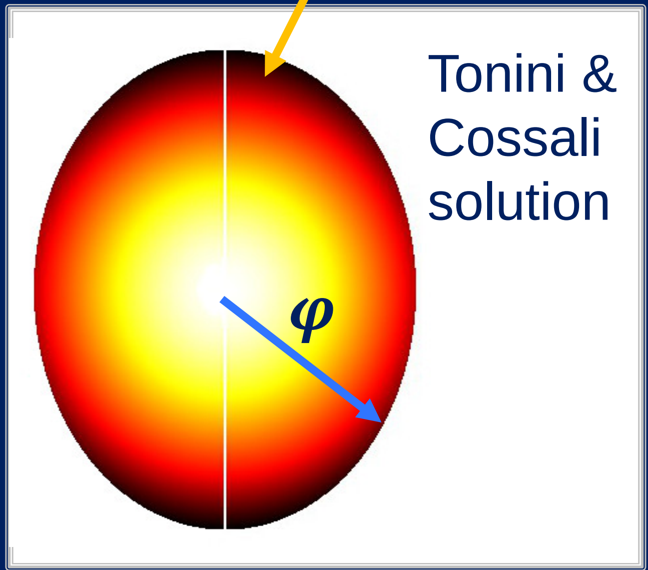
- Droplet shape changes (evaporation), but remains spheroidal

- Temperature is not constant inside and on the surface of the droplet



Mathematical Model

T– temperature:



Heat capacity

$$\rho C_p \frac{\partial T}{\partial t} = \nabla(k\nabla T)$$

Droplet density

Thermal conductivity

Boundary conditions at the drop surface:

$$-\mathbf{n}(-\mathrm{k}\nabla T) = \mathrm{h}\left(T_g - T\right) - q$$

Evaporation

Normal to the surface

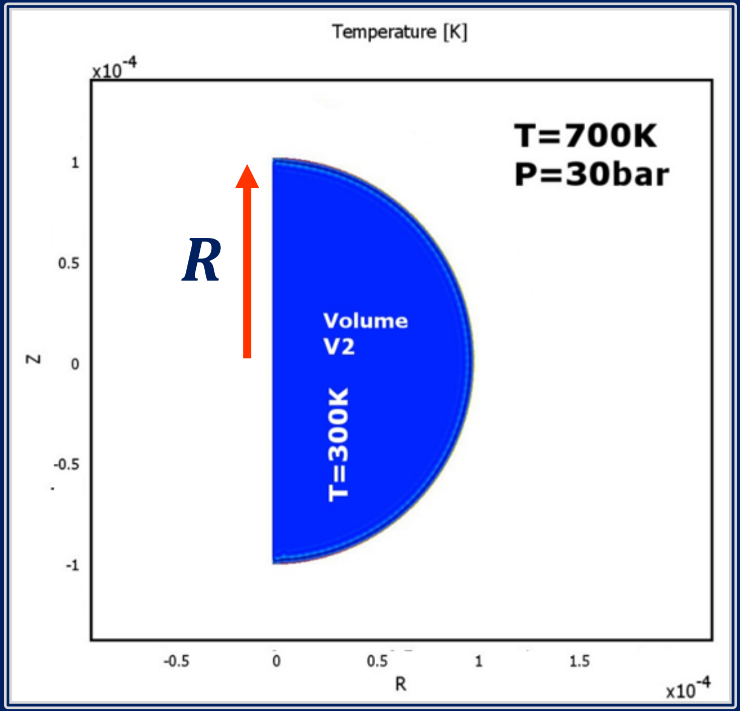
Heat transfer coefficient

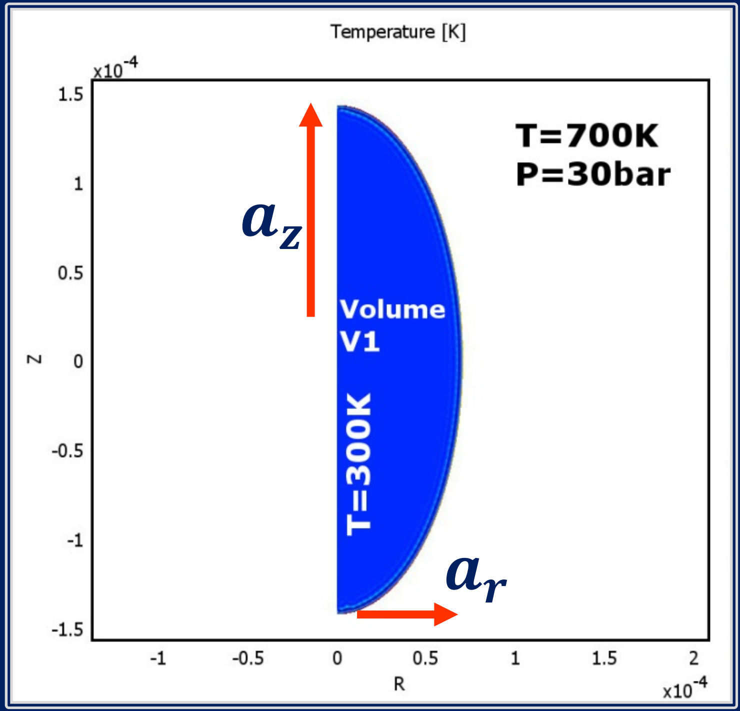
Latent Heat of vaporisation

$$\dot{\boldsymbol{\varphi}} = -U(T, x, y, z)$$

Defined by S. Tonini, G.E. Cossali (2014) solution for the gas phase

Model parameters



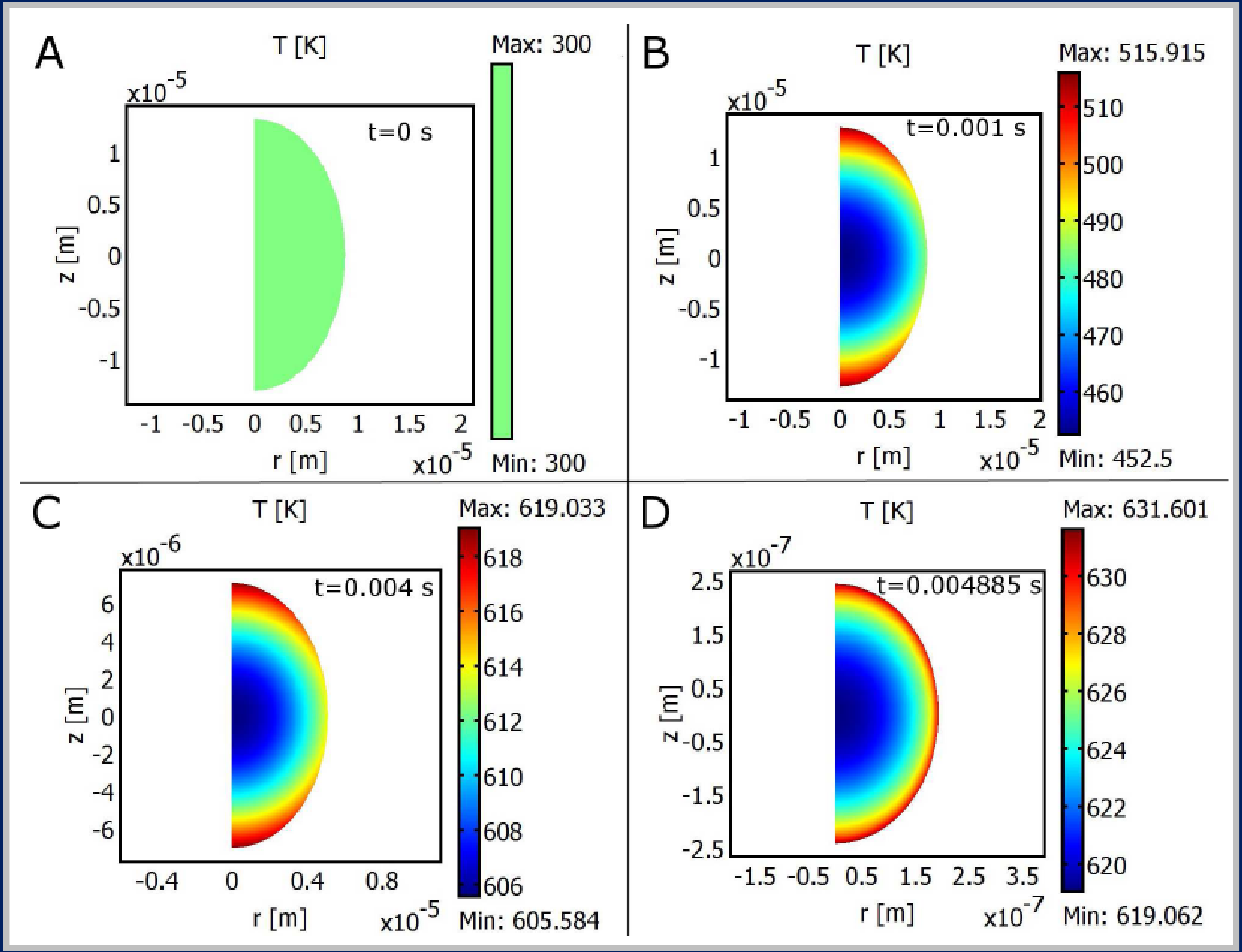


$$V_1 = V_2$$

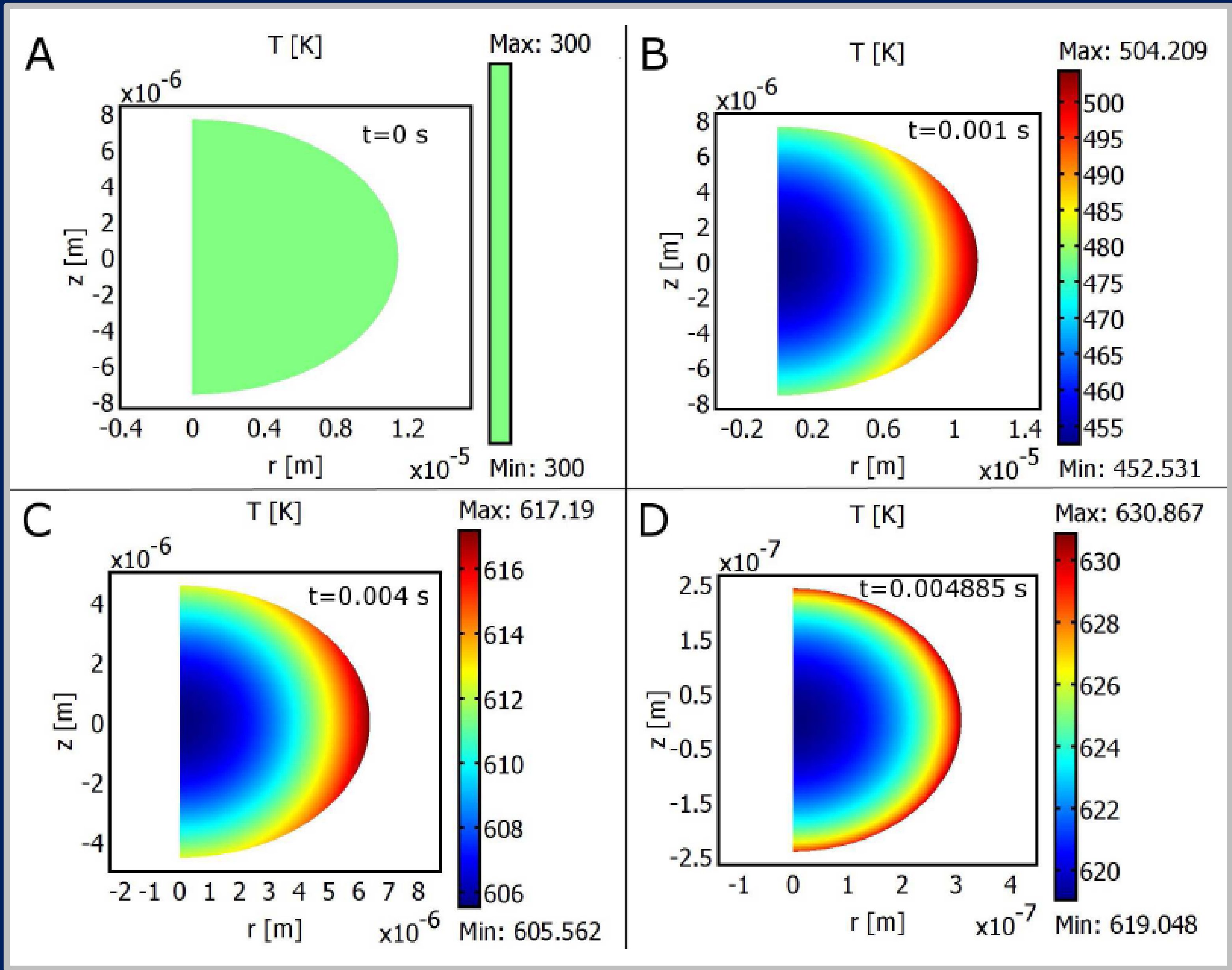
$$a_z/a_r = 1.5$$

$$R = 10\mu m$$

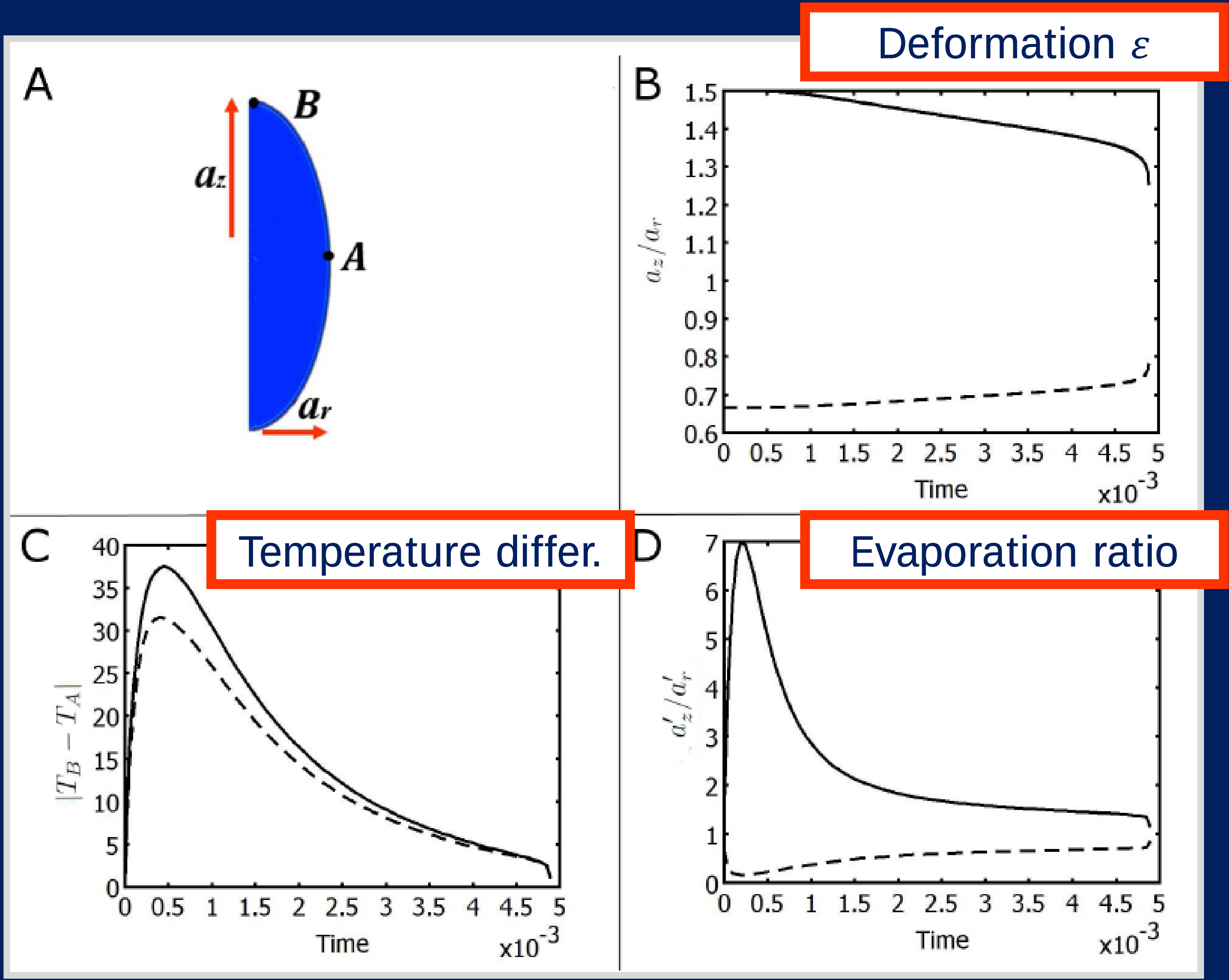
Results: Prolate droplet



Results: Oblate droplet



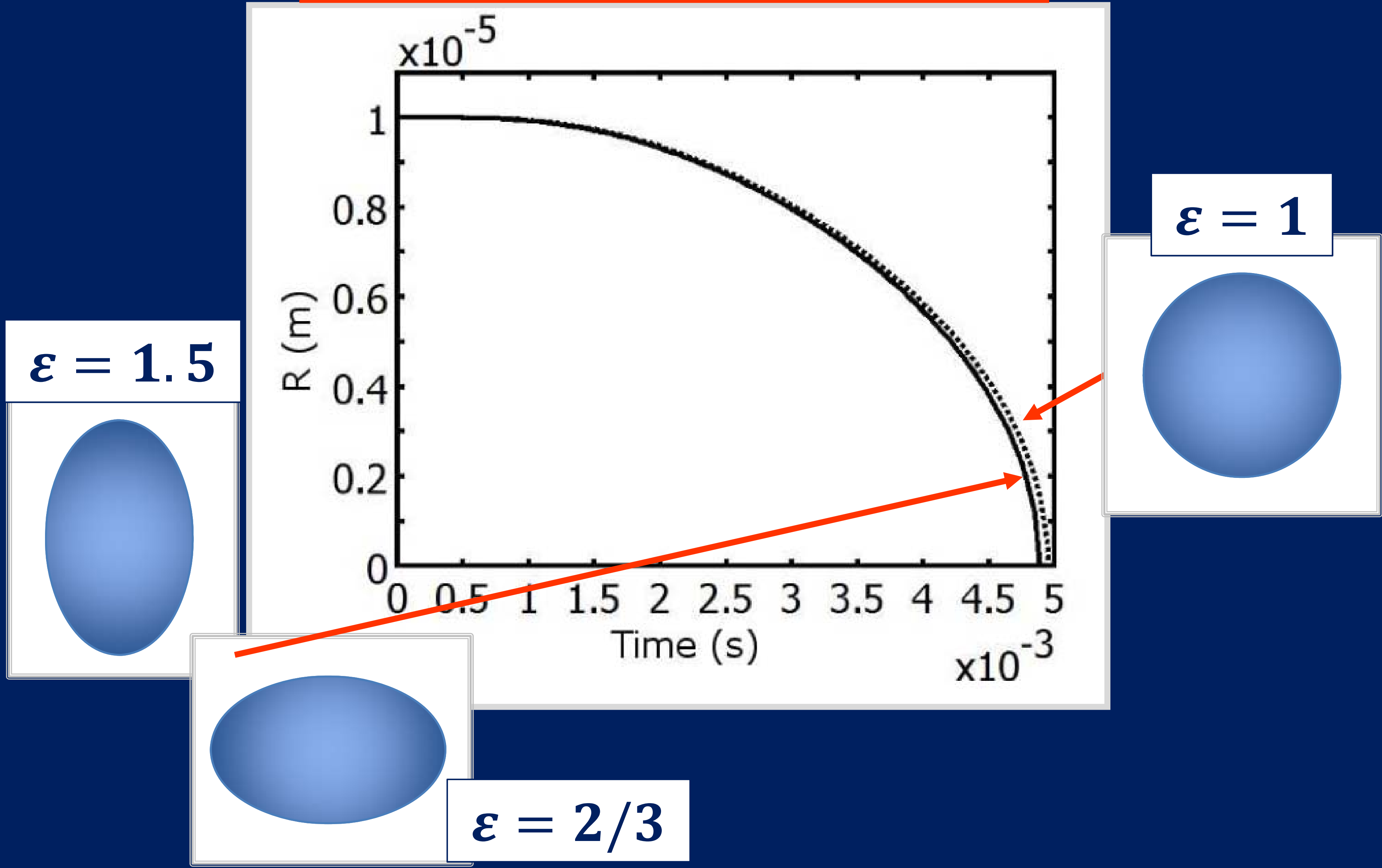
Results



ε is constant if surface *T* is uniform

Results. Droplet evaporation time

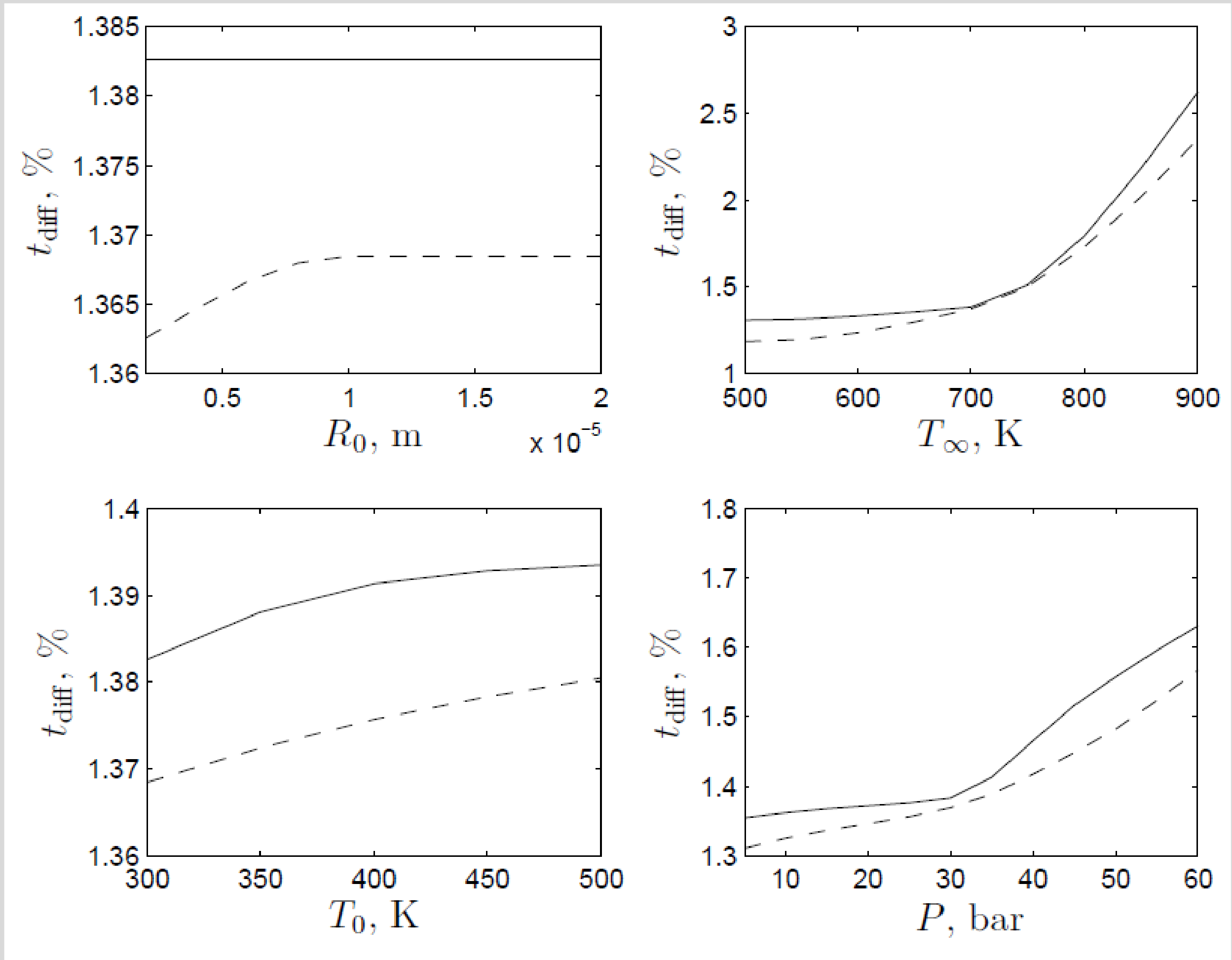
Effective droplet radius



Parameter sensitivity analysis

Relative difference of the droplet evaporation time for spherical and deformed droplets

$$t_{\mathrm{diff}} = (t_{\mathrm{sph}} - t_{\mathrm{def}})/t_{\mathrm{sph}} \cdot 100\%$$



Conclusion

- Local temperatures can vary noticeably along the droplet surface.
- Droplet heating is shown to be more intense in the regions with greatest curvature.
- Droplet becomes more spherical.
- The effect of droplet non-sphericity on the evaporation time of droplets was shown to be relatively small for the range of parameter values under consideration.

Acknowledgements

The authors are grateful to EPSRC (UK) (Project EP/K020528/1) for the financial support of this project.

University of Brighton

Advanced Engineering Centre