Mathematical Modelling of Heating and Evaporation of a Spheroidal Droplet

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Current mathematical models of sprays use spherical approximation for droplets.
Problem Statement

Current mathematical models of sprays use spherical approximation for droplets.
Can the spherical approximation of droplets be used?

We will consider heating and evaporation of a single non-spherical droplet and compare it with a spherical one.
Problem Statement

Heating and evaporation of a single non-spherical droplet.
Mathematical Modelling

Droplet dynamics:
Phase-field method – liquid and gas

Vapour dynamics:
Mixture Model of two phases – vapour and air

It is hard to couple these two models: we have three phases
Model simplifications

1. Approximate droplets by spheroids: 3D => 2D

2. Find temperature and vapour distribution for a steady state assuming that droplet shape is fixed and droplet temperature at the surface is uniform.

3. Use exact solution from for vapour in the boundary condition to calculate the reassertion of droplet surface and distribution of T inside the droplet.
Additional simplifications

We consider only slightly deformed droplets to guaranty that temperature gradient in radial direction is much larger than in the tangential direction.

We ignore:

• Droplet swelling (changes of the droplet volume due to changes in temperature);
• Surface tension and resulting oscillation of the droplet;
S. Tonini, G.E. Cossali (2014) proposed a mathematical model for non-spherical droplets:

Approximations:

- Problem is solved outside the droplet
- Droplet shape is fixed
- Temperature is constant on the droplet surface

\[ \varepsilon = \frac{a_z}{a_r} \]
The steady-state evaporation of a spheroidal drop can be analysed through the solution to the species balance equations:

\[ \nabla_j (\rho U \cdot \phi^\alpha - \rho D_v \nabla_j \phi^\alpha) = 0 \]

Species (air or vapour)

Mass diffusivity

Air and vapour can move due to convection and diffusion

Mass fraction

Total density

\[ \phi^\alpha = \frac{\rho^\alpha}{\rho} \]
In the spheroidal coordinate system the problem was solved analytically:

Evaporation rate of the droplet:

\[ m_{ev} = C(\varepsilon) \ln \frac{1}{1 - \varphi_s} \]

Evaporation factor

\[ C(\varepsilon) = 1, \text{ at } \varepsilon = 1 \]

Mass fraction

\[ \varepsilon = \frac{a_z}{a_r} \]
Tonini & Cossali Results

Non-spherical droplets evaporates faster.

Non-physical approximations:
- Droplet shape is fixed
- Temperature is constant inside and at the surface of the droplet
Model Development

Approximations:

• S. Tonini, G.E. Cossali (2014) mathematical model for non-spherical droplet for the gas phase outside the droplet

• Droplet shape changes (evaporation), but remains spheroidal

• Temperature is not constant inside and on the surface of the droplet
Mathematical Model

T- temperature:

\[ \phi = -U(T, x, y, z) \]

Heat capacity
\[ \rho C_p \frac{\partial T}{\partial t} = \nabla (k \nabla T) \]

Droplet density

Thermal conductivity

Boundary conditions at the drop surface:

Evaporation

Normal to the surface

Heat transfer coefficient

Latent Heat of vaporisation

Defined by S. Tonini, G.E. Cossali (2014) solution for the gas phase
Model parameters

$V_1 = V_2$

$\frac{a_z}{a_r} = 1.5$

$R = 10 \mu m$
Results: Prolate droplet

The images show the temperature distribution over time for a prolate droplet. Each panel represents a different time point and temperature range:

- Panel A: $t = 0$ s, temperature range $300-515.915$ K.
- Panel B: $t = 0.001$ s, temperature range $300-619.033$ K.
- Panel C: $t = 0.004$ s, temperature range $300-631.601$ K.
- Panel D: $t = 0.004885$ s, temperature range $300-619.062$ K.
Results: Oblate droplet
Results

Deformation $\varepsilon$

Temperature differ.

Evaporation ratio

$\varepsilon$ is constant if surface $T$ is uniform
Results. Droplet evaporation time

Effective droplet radius

$\varepsilon = 1.5$

$\varepsilon = 1$

$\varepsilon = 2/3$
Parameter sensitivity analysis

Relative difference of the droplet evaporation time for spherical and deformed droplets

\[ t_{\text{diff}} = \frac{(t_{\text{sph}} - t_{\text{def}})}{t_{\text{sph}}} \times 100\% \]
Conclusion

• Local temperatures can vary noticeably along the droplet surface.

• Droplet heating is shown to be more intense in the regions with greatest curvature.

• Droplet becomes more spherical.

• The effect of droplet non-sphericity on the evaporation time of droplets was shown to be relatively small for the range of parameter values under consideration.

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