Research Workshop, 16th August, 2013

Numerical modelling of two-phase vortex ring flow

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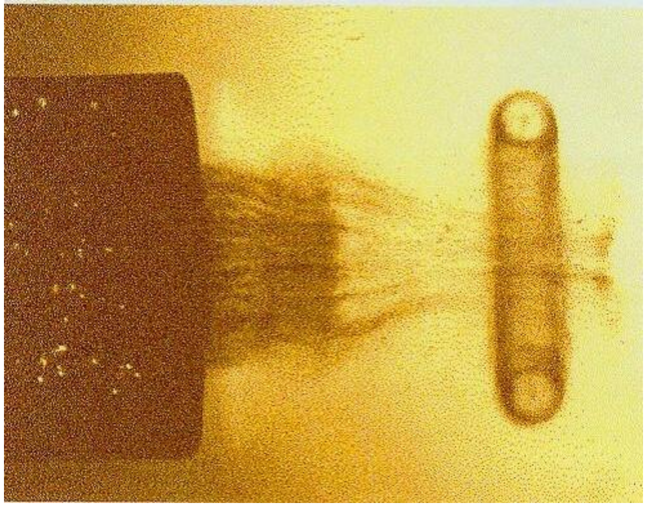
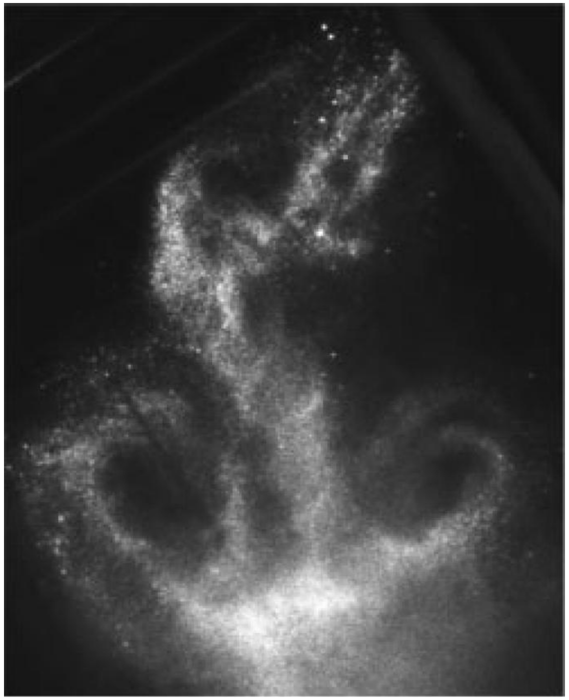
Outline

- Introduction
- Problem formulation
- Vortex ring flow
- Two-phase flow

Motivation



Formation of a Vortex Ring exiting a gun muzzle as captured by Dr. Lyons with spark photography upon firing a blank 40mm grenade (1997)



A typical high-speed photograph of a G-DI spray (Begg et al 2009)

EPSRC project “Development of the full Lagrangian approach for the analysis of vortex ring-like structures in disperse media: application to gasoline engines”

Introduction

- Vortex rings

(Helmholtz 1858; Lamb 1932; Phillips 1956; Norbury 1973; Kambe & Oshima 1975; Saffman 1992; Shariff & Leonard 1992; Lim & Nickels 1995; Stanaway, Cantwell & Spalart 1988; Rott & Cantwell 1993; Mohseni & Gharib 1998; Kaplanski & Rudi 1999, 2005; Fukumoto & Moffatt 2000; Shusser & Gharib 2000; Mohseni 2001, 2006; Linden & Turner 2001; Fukumoto & Kaplanski 2008, Kaplanski et al 2009, Kaplanski, Fukumoto & Rudi 2012)

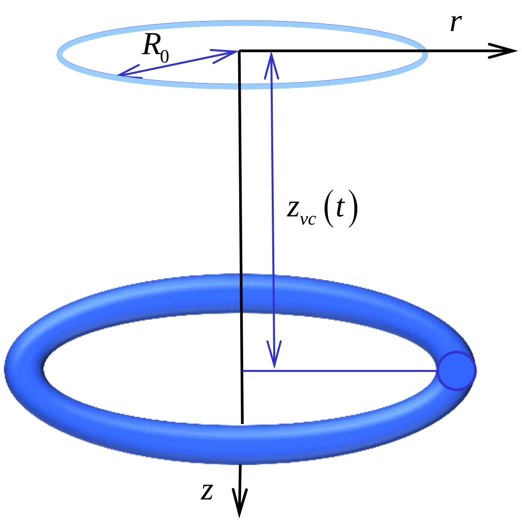
- Full Lagrangian Approach (FLA, Osiptov method) for particle motion simulation

Problem formulation

- One-way coupled two-fluid approach
  - Carrier phase: air (incompressible viscous fluid)
  - Dispersed phase: droplets of iso-octane (identical particles, pressureless continuum) of 10-50 μm in radius
- Force acting on a single particle:

$$\mathbf{f}_s = 6\pi\sigma\mu\left(\mathbf{v} - \mathbf{v}_s\right) + m\mathbf{g}$$

Vortex ring: formulation



- Axially symmetric flow

$$\frac{\partial \zeta}{\partial t}+\frac{\partial (u\zeta)}{\partial r}+\frac{\partial (v\zeta)}{\partial z}=\frac{1}{\mathrm{Re}_0}\left(\frac{\partial^2 \zeta}{\partial r^2}+\frac{\partial^2 \zeta}{\partial z^2}+\frac{1}{r}\frac{\partial \zeta}{\partial r}-\frac{\zeta}{r^2}\right)$$

$$t=0: \quad \zeta=\delta(z)\delta(r-1)$$

$$\zeta=\mathrm{rot}\,\mathbf{v}$$

$$\mathbf{r}=(r,z),\ \mathbf{v}=(u,v)$$

- Scale:

$$L=R_0, U=\frac{\Gamma_0}{R_0}, T=\frac{R_0^2}{\Gamma_0}$$

- Dimensionless parameter:

$$\mathrm{Re}_0=\frac{\Gamma_0}{\nu}$$

Vortex ring: solution (Fukumoto & Kaplanski, 2008)

- New variables:

$$\theta = \left(b\,\mathrm{Re}_0\right)^b t^{-b} = \left(\frac{b\,\mathrm{Re}_0}{t}\right)^b,$$
$$\xi = r\theta,$$
$$\eta = \left(z - z_{vc}(t)\right)\theta,$$
$$\zeta = \theta^3\omega$$

- Streamline function and velocity of the vortex centroid:

$$u = -\frac{1}{r}\frac{\partial\Psi}{\partial z}$$
$$v = \frac{1}{r}\frac{\partial\Psi}{\partial r} + V_{vc} = \frac{1}{r}\frac{\partial\Psi}{\partial r} + \dot{z}_{vc}(t)$$

$$V_{vc} = \frac{dz_{vc}}{dt}$$
$$z_{vc}(0) = 0$$

$$z_{vc} = \frac{2\pi\int_0^{\infty}\int_{-\infty}^{\infty} rz\zeta\,dz\,dr}{2\pi\int_0^{\infty}\int_{-\infty}^{\infty} r\zeta\,dz\,dr}$$

$$-\theta^{1/b-2}\left(3\omega + \theta\frac{\partial\omega}{\partial\theta} + \xi\frac{\partial\omega}{\partial\xi} + \eta\frac{\partial\omega}{\partial\eta}\right) + \theta\,\mathrm{Re}_0\left(\frac{\partial}{\partial\xi}\left(-\frac{\omega}{\xi}\frac{\partial\Psi}{\partial\eta}\right) + \frac{\partial}{\partial\eta}\left(\frac{\omega}{\xi}\frac{\partial\Psi}{\partial\xi}\right)\right) = \frac{\partial^2\omega}{\partial\xi^2} + \frac{\partial^2\omega}{\partial\eta^2} + \frac{1}{\xi}\frac{\partial\omega}{\partial\xi} - \frac{\omega}{\xi^2}$$

$$b = \frac{1}{2}, \quad \theta\,\mathrm{Re}_0 \ll 1$$

$$3\omega + \theta\frac{\partial\omega}{\partial\theta} + \xi\frac{\partial\omega}{\partial\xi} + \eta\frac{\partial\omega}{\partial\eta} = \frac{\partial^2\omega}{\partial\xi^2} + \frac{\partial^2\omega}{\partial\eta^2} + \frac{1}{\xi}\frac{\partial\omega}{\partial\xi} - \frac{\omega}{\xi^2}$$

Vortex ring: solution (Fukumoto & Kaplanski, 2008)

$$\zeta = \frac{1}{\sqrt{2\pi}}\theta^3 \exp\left(-\frac{\xi^2+\eta^2+\theta^2}{2}\right)\mathrm{I}_1(\xi\theta)$$

$$\Psi = \frac{1}{4}\xi\int_0^\infty F(x,\eta)\,\mathrm{J}_1(\theta x)\,\mathrm{J}_1(\xi x)\,dx$$

$$F(x,\eta) = \exp(x\eta)\,\mathrm{erfc}\left(\frac{x+\eta}{\sqrt{2}}\right) + \exp(-x\eta)\,\mathrm{erfc}\left(\frac{x-\eta}{\sqrt{2}}\right)$$

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty \exp(-y^2)\,dy$$

$$V_{vc} = \frac{1}{4\sqrt{\pi}}\theta\left[3\exp\left(-\frac{\theta^2}{2}\right)\mathrm{I}_1\left(\frac{\theta^2}{2}\right) + \frac{\theta^2}{12}\,{}_2F_2\left(\left\{\frac{3}{2},\frac{3}{2}\right\},\left\{\frac{5}{2},3\right\},-\theta^2\right) - \frac{3\theta^2}{5}\,{}_2F_2\left(\left\{\frac{3}{2},\frac{5}{2}\right\},\left\{2,\frac{7}{2}\right\},-\theta^2\right)\right]$$

$\theta \to 0\ (t \to \infty)$

$$V_{vc} = \frac{7}{120\sqrt{\pi}}\theta^3 + O(\theta^5)$$

$\theta \to \infty\ (t \to 0)$

$$V_{vc} \sim \frac{1}{8\pi}\left(2\ln\theta + 3 - \gamma - 2\psi(1.5)\right)$$

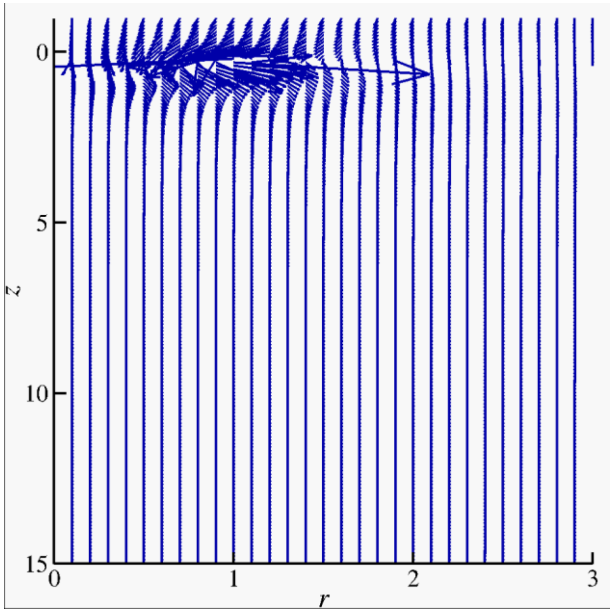
γ The Euler constant

ψ Digamma function

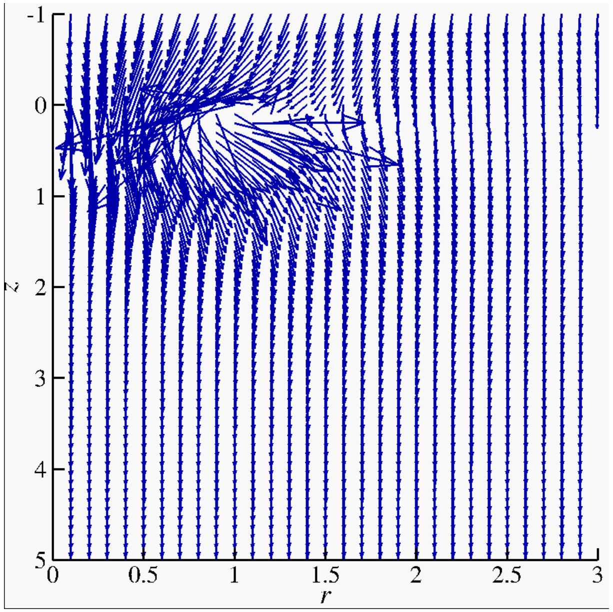
Vortex ring: solution

$$u = -\frac{1}{4}\theta^2 \int_0^\infty x\left(\exp(x\eta)\,\mathrm{erfc}\left(\frac{x+\eta}{\sqrt{2}}\right) - \exp(-x\eta)\,\mathrm{erfc}\left(\frac{x-\eta}{\sqrt{2}}\right)\right) \mathrm{J}_1(\theta x)\,\mathrm{J}_1(\xi x)\,dx$$

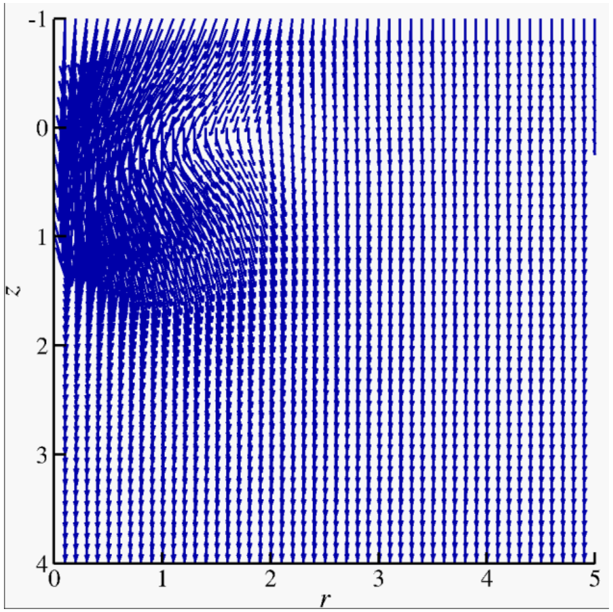
$$v = \frac{\theta^2}{4}\int_0^\infty xF(x,\eta)\,\mathrm{J}_1(\theta x)\,\mathrm{J}_0(\xi x)\,dx + V_{vc}$$



$\mathrm{Re}_0 = 2650$



$\mathrm{Re}_0 = 165.5$



$\mathrm{Re}_0 = 0.2$

Dispersed phase

$$\frac{dr_d}{dt} = u_d \qquad \frac{dz_d}{dt} = v_d$$

$$\frac{du_d}{dt} = \beta\left(u - u_d\right) \qquad \frac{dv_d}{dt} = \beta\left(v - v_d\right) + \frac{1}{\mathrm{Fr}^2}$$

$$n_s r\left|J\right| = n_{s0} r_0$$

$$\left|J\right| = \begin{vmatrix} \dfrac{\partial r_d}{\partial r_0} & \dfrac{\partial z_d}{\partial r_0} \\ \dfrac{\partial r_d}{\partial z_0} & \dfrac{\partial z_d}{\partial z_0} \end{vmatrix} \qquad q_{ij} = \frac{\partial v_{id}}{\partial x_{j0}}$$

$$\frac{\partial J_{ij}}{\partial t} = q_{ij} \qquad \frac{\partial q_{ij}}{\partial t} = \beta\left(\frac{\partial v_i}{\partial x_1} J_{1j} + \frac{\partial v_i}{\partial x_2} J_{2j} - q_{ij}\right)$$

$1 - r$

$2 - z$

Dimensionless parameters:

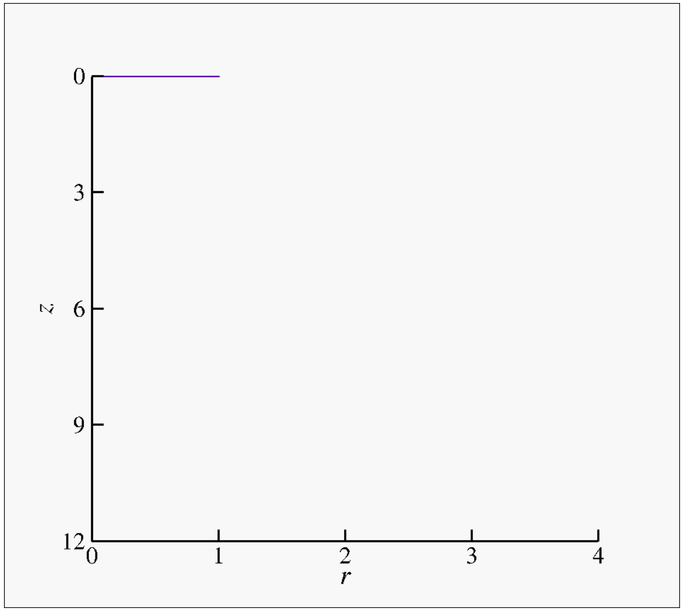
$$\beta = \frac{6\pi\sigma\mu R_0^2}{m\Gamma_0} \qquad \mathrm{Fr} = \frac{\Gamma_0}{R_0\sqrt{gR_0}}$$

Initial conditions:

$$r_d = r_{d0}, \quad z_d = z_{d0},$$
$$u_d = u_{d0}, \quad v_d = v_{d0},$$
$$q_{ij} = 0, \quad J_{ij} = \delta_{ij}$$

Droplets: numerical simulation

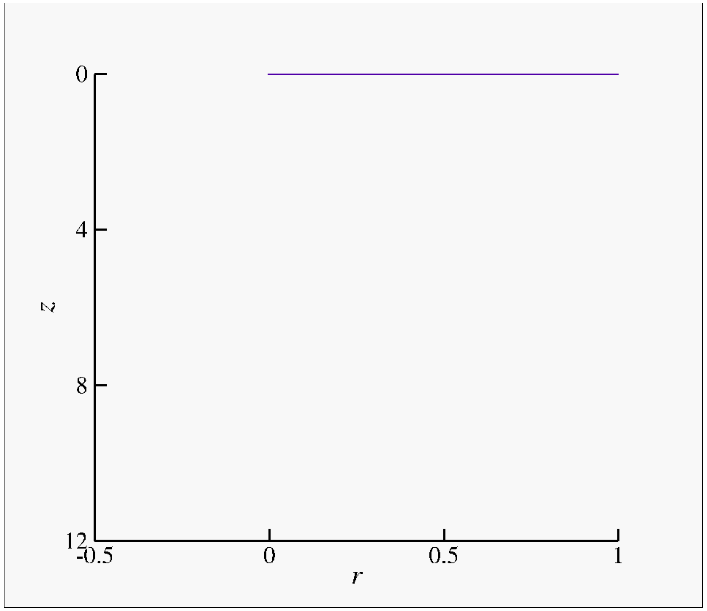
Particle motion: a line of particles



$\mathrm{Re}_0 = 165.5$

$\beta = 5.3$

$\mathrm{Fr} = 6.4$



$\mathrm{Re}_0 = 165.5$

$\beta = 0.2$

$\mathrm{Fr} = 6.4$

Initial conditions:

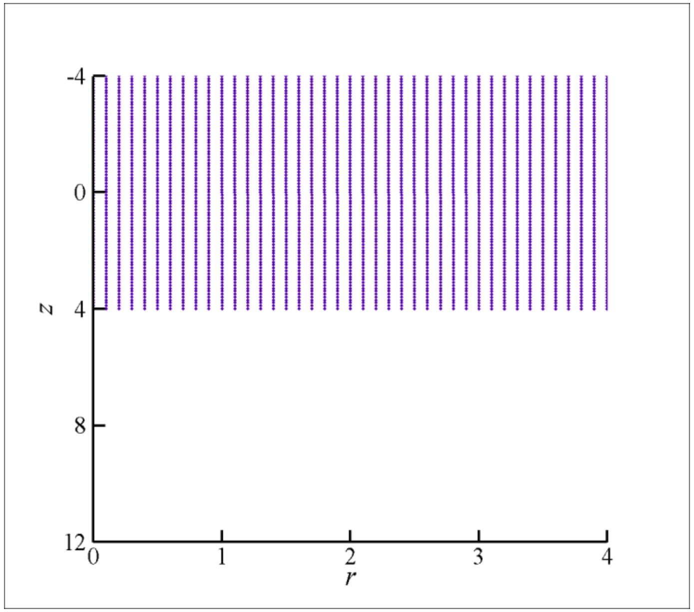
$t_0 = 0.2$

$r_d = r_{d0}, \quad z_d = 0,$

$u_d = 0, \quad v_d = 0$

Droplets: numerical simulation

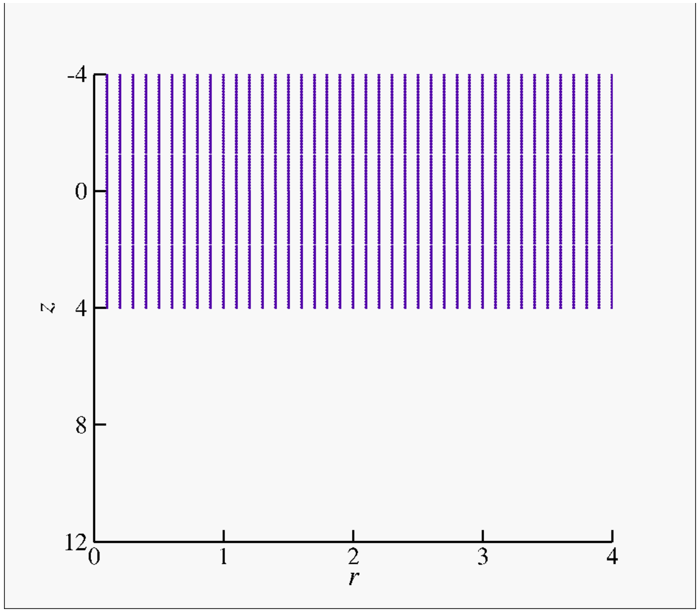
Particle motion: a cloud of particles



$\mathrm{Re}_0 = 165.5$

$\beta = 5.3$

$\mathrm{Fr} = 6.4$



$\mathrm{Re}_0 = 165.5$

$\beta = 0.2$

$\mathrm{Fr} = 6.4$

Initial conditions:

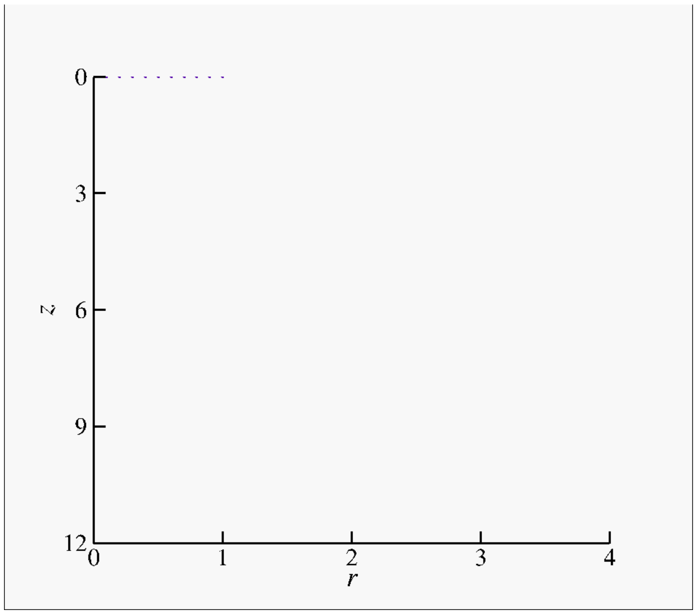
$t_0 = 0.2$

$r_d = r_{d0}, \quad z_d = z_{d0},$

$u_d = 0, \quad v_d = 0$

Droplets: numerical simulation

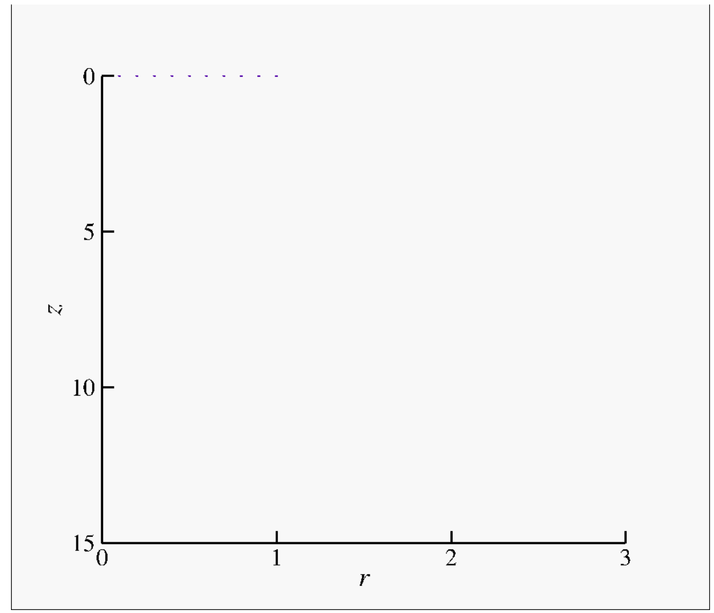
Particle motion: a cloud of particles



$\mathrm{Re}_0 = 165.5$

$\beta = 5.3$

$\mathrm{Fr} = 6.4$



$\mathrm{Re}_0 = 165.5$

$\beta = 0.2$

$\mathrm{Fr} = 6.4$

Initial conditions:

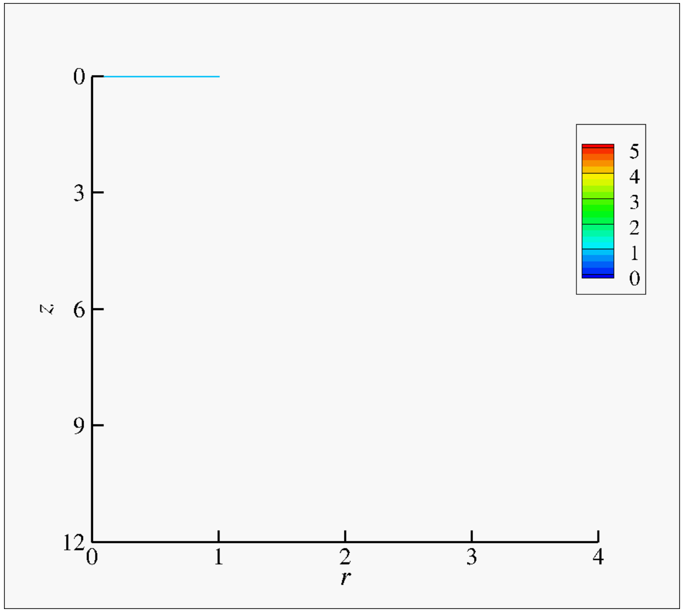
$t_0 \in [0.2, 1.2]$

$r_d = r_{d0}, \quad z_d = 0,$

$u_d = 0, \quad v_d = 0.6$

Droplets: numerical simulation

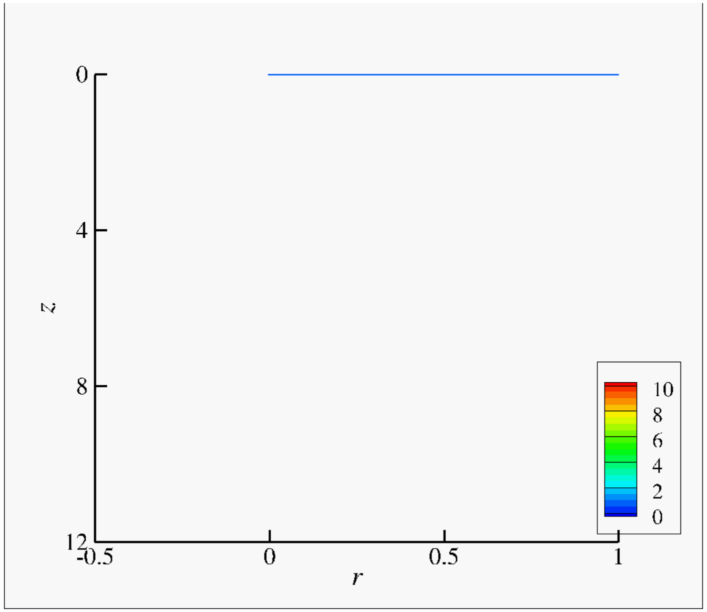
Particle concentration: a line of particles



$\mathrm{Re}_0 = 165.5$

$\beta = 5.3$

$\mathrm{Fr} = 6.4$



$\mathrm{Re}_0 = 165.5$

$\beta = 0.2$

$\mathrm{Fr} = 6.4$

Initial conditions:

$t_0 = 0.2$

$r_d = r_{d0}, \quad z_d = 0,$

$u_d = 0, \quad v_d = 0$

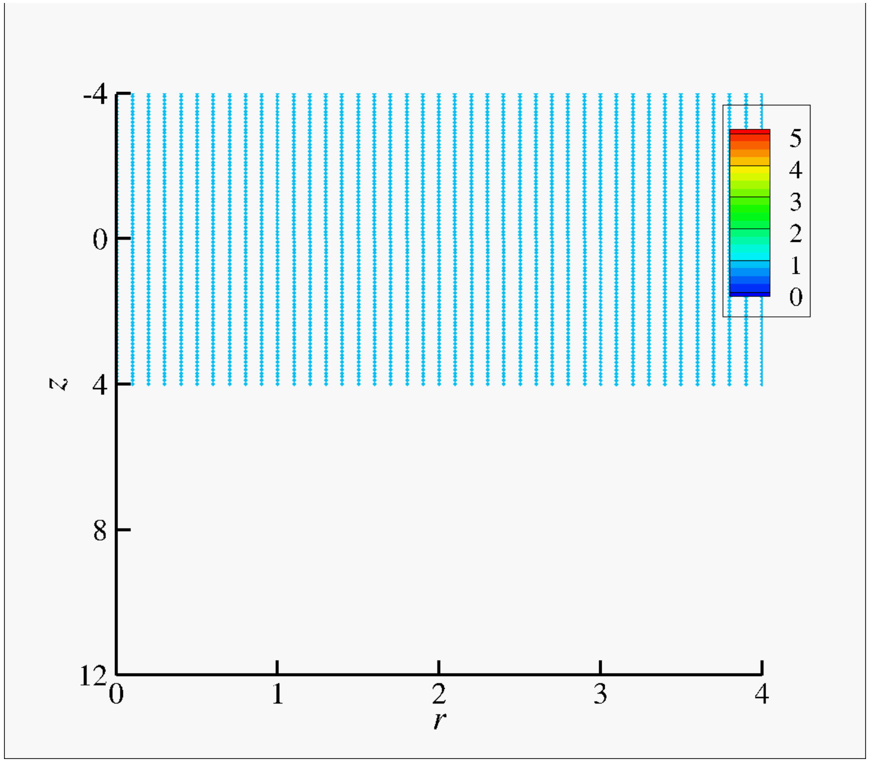
Droplets: numerical simulation

Particle concentration: a cloud of particles

$\mathrm{Re}_0 = 165.5$

$\beta = 0.2$

$\mathrm{Fr} = 6.4$



Initial conditions:

$t_0 = 0.2$

$r_d = r_{d0}, \quad z_d = 0,$

$u_d = 0, \quad v_d = 0$

Conclusions:

- Unsteady axially symmetric two phase vortex ring flow modelled using FLA and Kaplanski analytical solution
- Further investigation to compare numerical results with the experimental data is needed