Research Workshop, 20th November, 2013

Numerical modelling of a two-phase vortex ring flow

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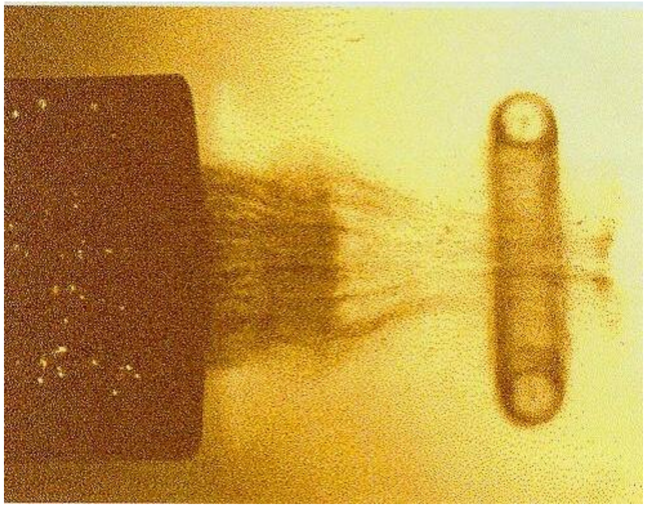
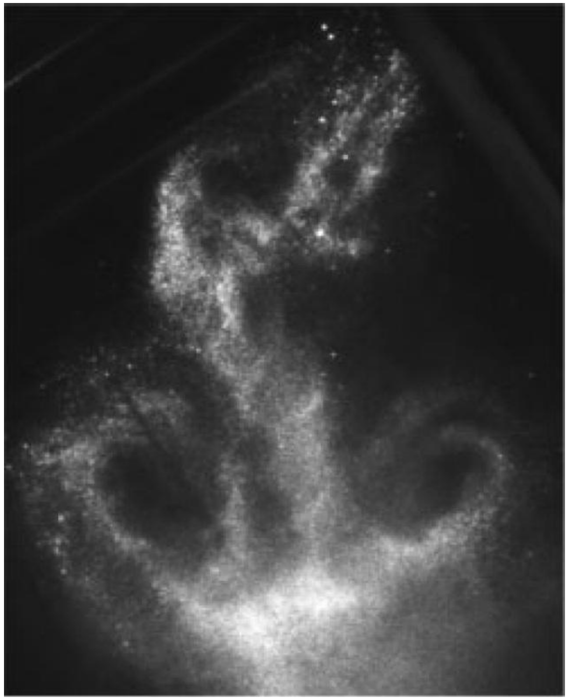
Outline

- Introduction
- Problem formulation
- Vortex ring flow
- Two-phase flow

Motivation



Formation of a Vortex Ring exiting a gun muzzle as captured by Dr. Lyons with spark photography upon firing a blank 40mm grenade (1997)



A typical high-speed photograph of a G-DI spray (Begg et al 2009)

<u>EPSRC project</u> "Development of the full Lagrangian approach for the analysis of vortex ring-like structures in disperse media: application to gasoline engines"

Introduction

- Vortex rings

(Helmholtz 1858; Lamb 1932; Phillips 1956; Norbury 1973; Kambe & Oshima 1975; Saffman 1992; Shariff & Leonard 1992; Lim & Nickels 1995; Stanaway, Cantwell & Spalart 1988; Rott & Cantwell 1993; Mohseni & Gharib 1998; Kaplanski & Rudi 1999, 2005; Fukumoto & Moffatt 2000; Shusser & Gharib 2000; Mohseni 2001, 2006; Linden & Turner 2001; Fukumoto & Kaplanski 2008, Kaplanski et al 2009, Kaplanski, Fukumoto & Rudi 2012)

- Full Lagrangian Approach (FLA, Osiptov method) for particle motion simulation

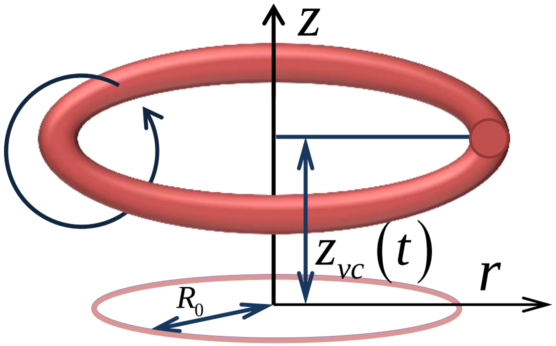
Problem formulation

- One-way coupled two-fluid approach
  - Carrier phase: air (incompressible viscous fluid)
  - Dispersed phase: droplets of iso-octane (identical particles, pressureless continuum) of 10-50 μm in radius
- Force acting on a single particle:

$$\mathbf{f}_s = 6\pi\sigma\mu\left(\mathbf{v} - \mathbf{v}_s\right)\left(1 + \frac{1}{6}\mathrm{Re}_s^{2/3}\right) + m\mathbf{g}$$

Vortex ring: formulation

$$\frac{\partial\zeta}{\partial t}+\frac{\partial(u\zeta)}{\partial r}+\frac{\partial(v\zeta)}{\partial z}+\frac{\zeta u}{r}=\frac{1}{\mathrm{Re}_0}\left(\frac{\partial^2\zeta}{\partial r^2}+\frac{\partial^2\zeta}{\partial z^2}+\frac{1}{r}\frac{\partial\zeta}{\partial r}-\frac{\zeta}{r^2}\right)$$



$$t=0: \quad \zeta=\delta(z)\delta(r-1)$$

$$r=0: \quad \zeta=0, \Psi=0$$

$$\sqrt{r^2+z^2}\to\infty: \quad \zeta\to 0, \Psi=0$$

- Axially symmetric flow

$$\zeta=\mathrm{rot}\,\mathbf{v}$$

$$\mathbf{r}=(r,z), \mathbf{v}=(u,v)$$

- Scales:

$$L=R_0, U=\frac{\Gamma_0}{R_0}, T=\frac{R_0^2}{\Gamma_0}$$

- Dimensionless parameter:

$$\mathrm{Re}_0=\frac{\Gamma_0}{\nu}$$

Vortex ring: solution (Fukumoto & Kaplanski, 2008)

- Streamline function and velocity of the vortex centroid:

$$u = \frac{1}{r}\frac{\partial \Psi}{\partial z}, \quad v = -\frac{1}{r}\frac{\partial \Psi}{\partial r}$$

$$\Psi = -\frac{r\sqrt{\mathrm{Re}_0}}{4\sqrt{2t}} \int_0^\infty \left(\begin{aligned} &\exp\left(\sqrt{\mathrm{Re}_0}\,\frac{x\left(z - z_{vc}\right)}{\sqrt{2t}}\right)\mathrm{erfc}\left(\frac{x}{\sqrt{2}} + \sqrt{\mathrm{Re}_0}\,\frac{z - z_{vc}}{2\sqrt{t}}\right) + \\ &+\exp\left(-\sqrt{\mathrm{Re}_0}\,\frac{x\left(z - z_{vc}\right)}{\sqrt{2t}}\right)\mathrm{erfc}\left(\frac{x}{\sqrt{2}} - \sqrt{\mathrm{Re}_0}\,\frac{z - z_{vc}}{2\sqrt{t}}\right) \end{aligned} \right) \mathrm{J}_1\left(\sqrt{\mathrm{Re}_0}\,\frac{x}{\sqrt{2t}}\right)\mathrm{J}_1\left(\sqrt{\mathrm{Re}_0}\,\frac{rx}{\sqrt{2t}}\right)dx$$

- Velocity of the vortex centroid:

$$V_{vc} = \frac{dz_{vc}}{dt} \qquad z_{vc}\left(0\right) = 0$$

$$V_{vc} = \frac{\sqrt{\mathrm{Re}_0}}{4\sqrt{2\pi}}t^{-1/2}\left[3\exp\left(-\frac{\mathrm{Re}_0}{4t}\right)\mathrm{I}_1\left(\frac{\mathrm{Re}_0}{4t}\right) + \frac{\mathrm{Re}_0}{24t}\,{}_2F_2\left(\left\{\frac{3}{2},\frac{3}{2}\right\},\left\{\frac{5}{2},3\right\},-\frac{\mathrm{Re}_0}{2t}\right) - \frac{3\mathrm{Re}_0}{10t}\,{}_2F_2\left(\left\{\frac{3}{2},\frac{5}{2}\right\},\left\{2,\frac{7}{2}\right\},-\frac{\mathrm{Re}_0}{2t}\right)\right]$$

Vortex ring: solution (Fukumoto & Kaplanski, 2008)

- Velocity of the vortex centroid:

$$V_{vc} = \frac{\sqrt{\mathrm{Re}_0}}{4\sqrt{2\pi}} t^{-1/2} \left[3\exp\left(-\frac{\mathrm{Re}_0}{4t}\right) \mathrm{I}_1\left(\frac{\mathrm{Re}_0}{4t}\right) + \frac{\mathrm{Re}_0}{24t}\, {}_2F_2\left(\left\{\frac{3}{2},\frac{3}{2}\right\},\left\{\frac{5}{2},3\right\},-\frac{\mathrm{Re}_0}{2t}\right) - \frac{3\mathrm{Re}_0}{10t}\, {}_2F_2\left(\left\{\frac{3}{2},\frac{5}{2}\right\},\left\{2,\frac{7}{2}\right\},-\frac{\mathrm{Re}_0}{2t}\right)\right]$$

$$t \to 0$$

$$V_{vc} \sim \frac{1}{8\pi}\left(\ln\left(\frac{\mathrm{Re}}{2t}\right) + 3 - \gamma - 2\psi(1.5)\right)$$

$$t \to \infty$$

$$V_{vc} = \frac{7}{120\sqrt{\pi}}\left(\frac{\mathrm{Re}}{2t}\right)^{3/2} + O\left(\left(\frac{\mathrm{Re}}{2t}\right)^{5/2}\right)$$

γ – The Euler-Mascheroni constant

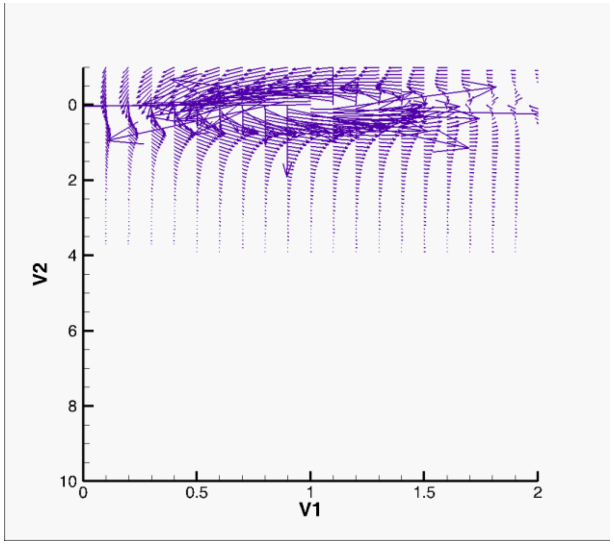
ψ – Digamma function

Vortex ring: solution

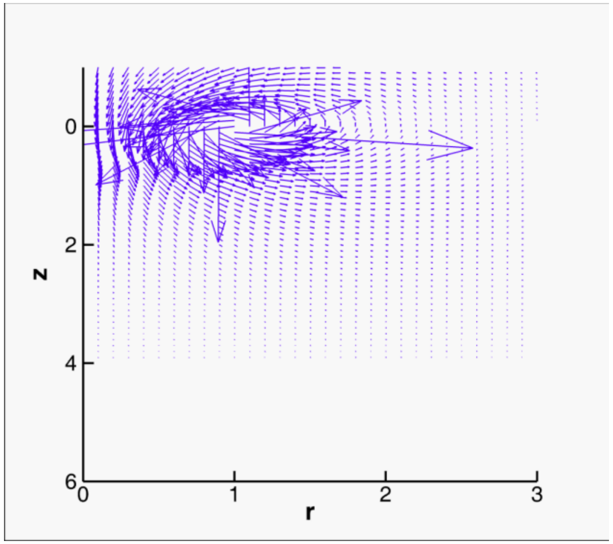
$$u=-\frac{\mathrm{Re}_0}{8t}\left[\int_0^\infty x\left(\begin{array}{l}\mathrm{erfc}\left(\frac{x}{\sqrt{2}}+\sqrt{\mathrm{Re}_0}\,\frac{z-z_{vc}}{2\sqrt{t}}\right)\exp\left(\sqrt{\mathrm{Re}_0}\,\frac{x\left(z-z_{vc}\right)}{\sqrt{2t}}\right)-\\-\mathrm{erfc}\left(\frac{x}{\sqrt{2}}-\sqrt{\mathrm{Re}_0}\,\frac{z-z_{vc}}{2\sqrt{t}}\right)\exp\left(-\sqrt{\mathrm{Re}_0}\,\frac{x\left(z-z_{vc}\right)}{\sqrt{2t}}\right)\end{array}\right)\mathrm{J}_1\left(\sqrt{\mathrm{Re}_0}\,\frac{x}{\sqrt{2t}}\right)\mathrm{J}_1\left(\sqrt{\mathrm{Re}_0}\,\frac{rx}{\sqrt{2t}}\right)dx\right]$$

$$v=\frac{\mathrm{Re}_0}{8t}\int_0^\infty x\left(\begin{array}{l}\exp\left(\sqrt{\mathrm{Re}_0}\,\frac{x\left(z-z_{vc}\right)}{\sqrt{2t}}\right)\mathrm{erfc}\left(\frac{x}{\sqrt{2}}+\sqrt{\mathrm{Re}_0}\,\frac{z-z_{vc}}{2\sqrt{t}}\right)+\\+\exp\left(-\sqrt{\mathrm{Re}_0}\,\frac{x\left(z-z_{vc}\right)}{\sqrt{2t}}\right)\mathrm{erfc}\left(\frac{x}{\sqrt{2}}-\sqrt{\mathrm{Re}_0}\,\frac{z-z_{vc}}{2\sqrt{t}}\right)\end{array}\right)\mathrm{J}_1\left(\sqrt{\mathrm{Re}_0}\,\frac{x}{\sqrt{2t}}\right)\mathrm{J}_0\left(\sqrt{\mathrm{Re}_0}\,\frac{rx}{\sqrt{2t}}\right)dx$$

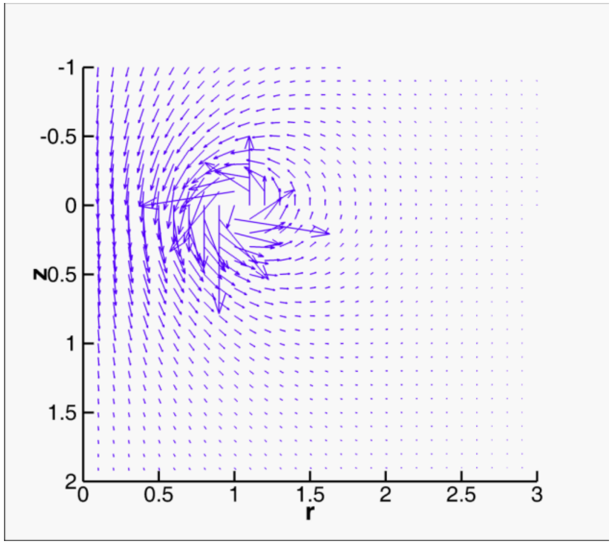
Vortex ring: solution



$\mathrm{Re}_0 = 1000$



$\mathrm{Re}_0 = 100$



$\mathrm{Re}_0 = 10$

Dispersed phase

Momentum balance equations:

$$\frac{dr_d}{dt} = u_d \qquad \frac{dz_d}{dt} = v_d$$

$$\frac{du_d}{dt} = \beta\left(u - u_d\right)\left(1 + \frac{1}{6}\mathrm{Re}_s^{2/3}\right) \qquad \frac{dv_d}{dt} = \beta\left(v - v_d\right)\left(1 + \frac{1}{6}\mathrm{Re}_s^{2/3}\right) + \frac{1}{\mathrm{Fr}^2}$$

$$\mathrm{Re}_s = \mathrm{Re}_{s0}\left|\mathbf{v} - \mathbf{v}_s\right|$$

Dimensionless parameters:

$$\beta = \frac{6\pi\sigma\mu R_0^2}{m\Gamma_0} \qquad \mathrm{Fr} = \frac{\Gamma_0}{R_0\sqrt{gR_0}} \qquad \mathrm{Re}_{s0} = \frac{2\sigma\Gamma_0}{R_0\nu} \qquad \mathrm{Re}_s = \mathrm{Re}_{s0}\left|\mathbf{v} - \mathbf{v}_s\right|$$

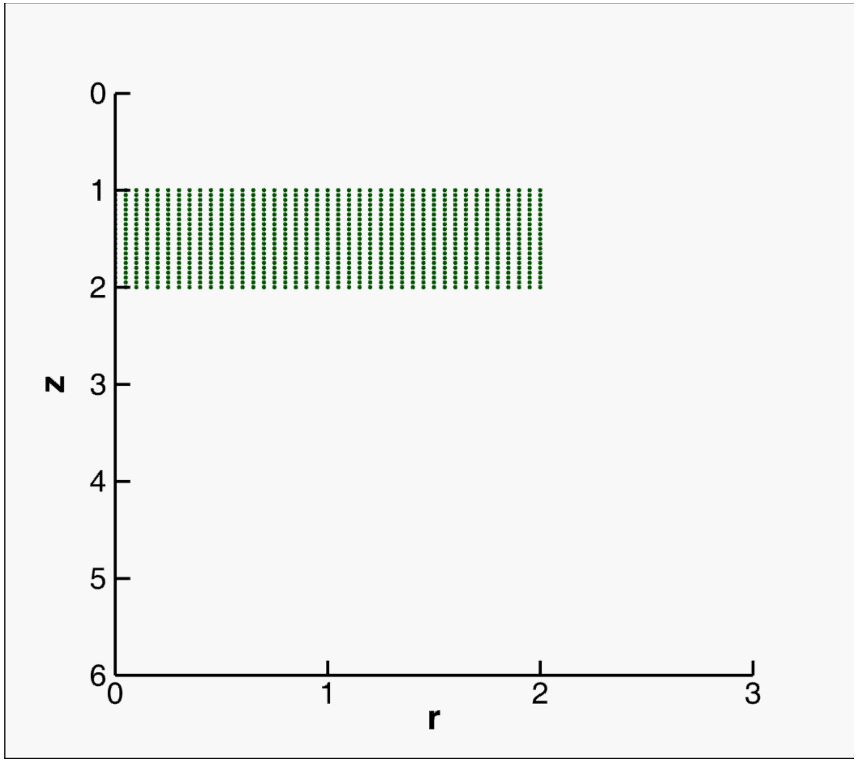
Initial conditions:

$$r_d = r_{d0}, \quad z_d = z_{d0},$$

$$u_d = u_{d0}, \quad v_d = v_{d0}$$

Droplets: numerical simulation

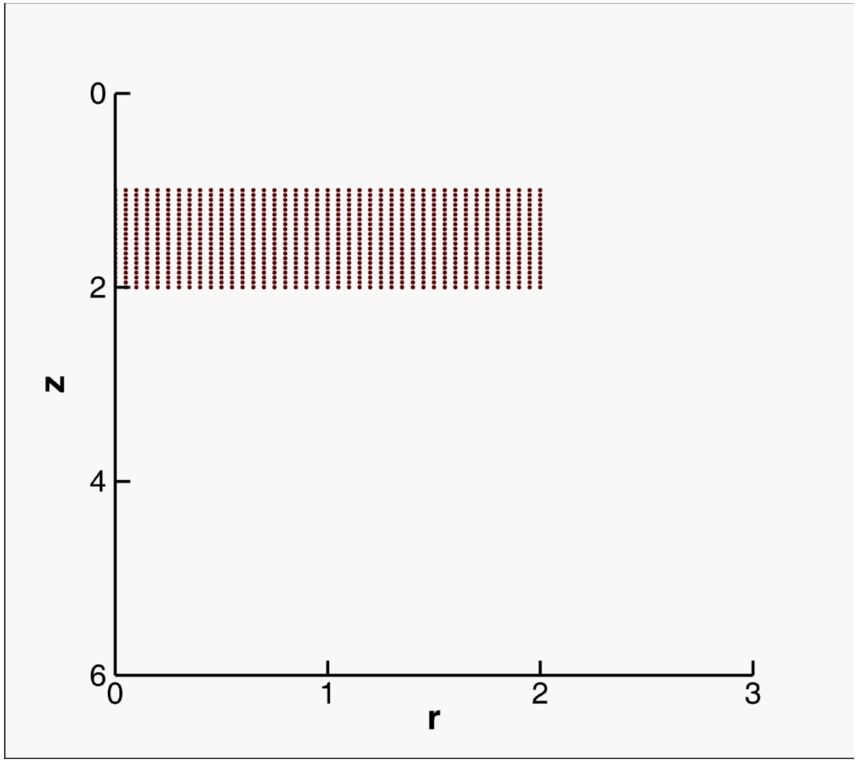
A dust cloud ahead of the vortex ring, Re = 100



$\beta = 3$

$\mathrm{Fr} = 7.5$

$\mathrm{Re}_{s0} = 0.7$



$\beta = 0.13$

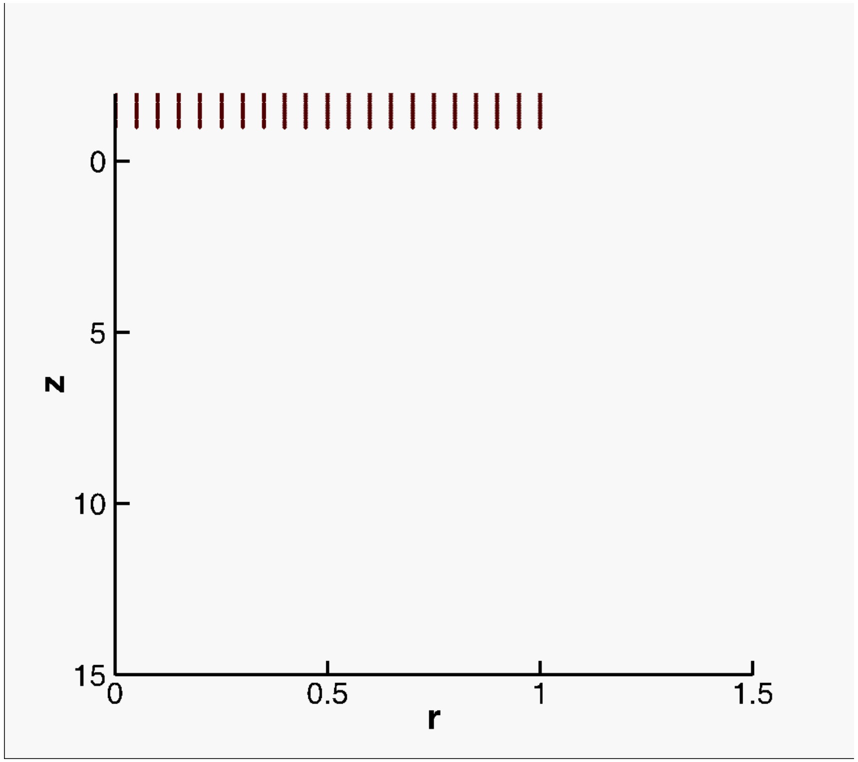
$\mathrm{Fr} = 7.5$

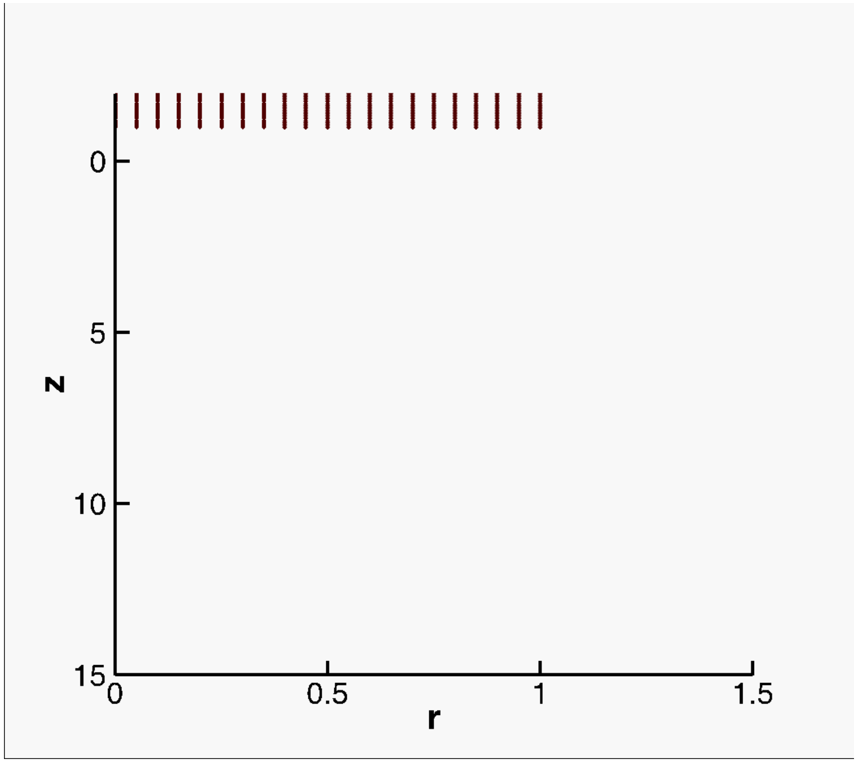
$\mathrm{Re}_{s0} = 3.3$

Droplets: numerical simulation

A dust cloud behind the vortex ring, Re = 100

$$\sigma = 50 \mu m$$

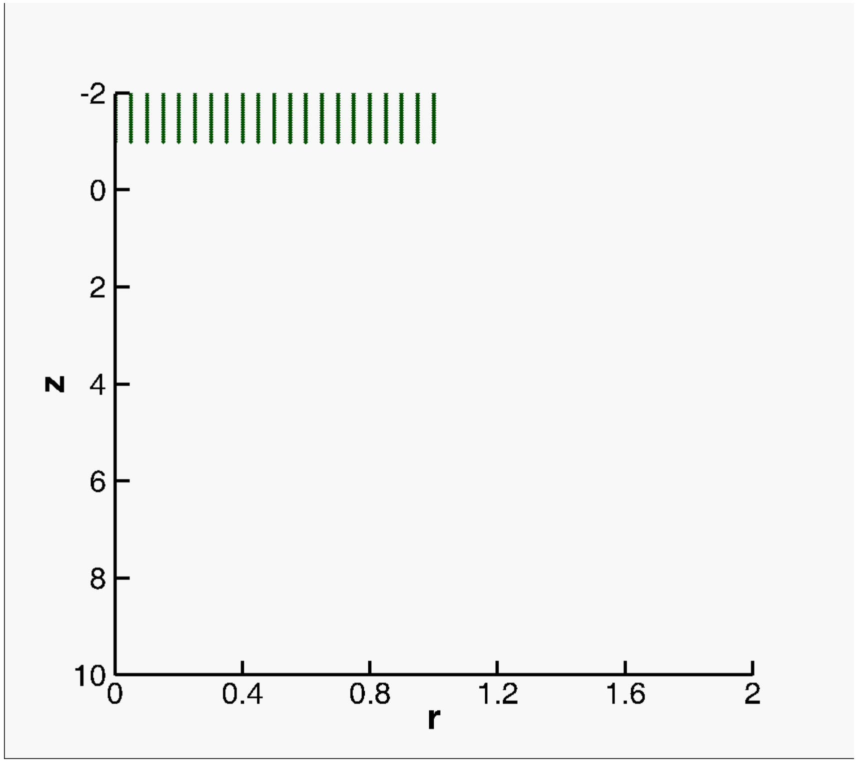


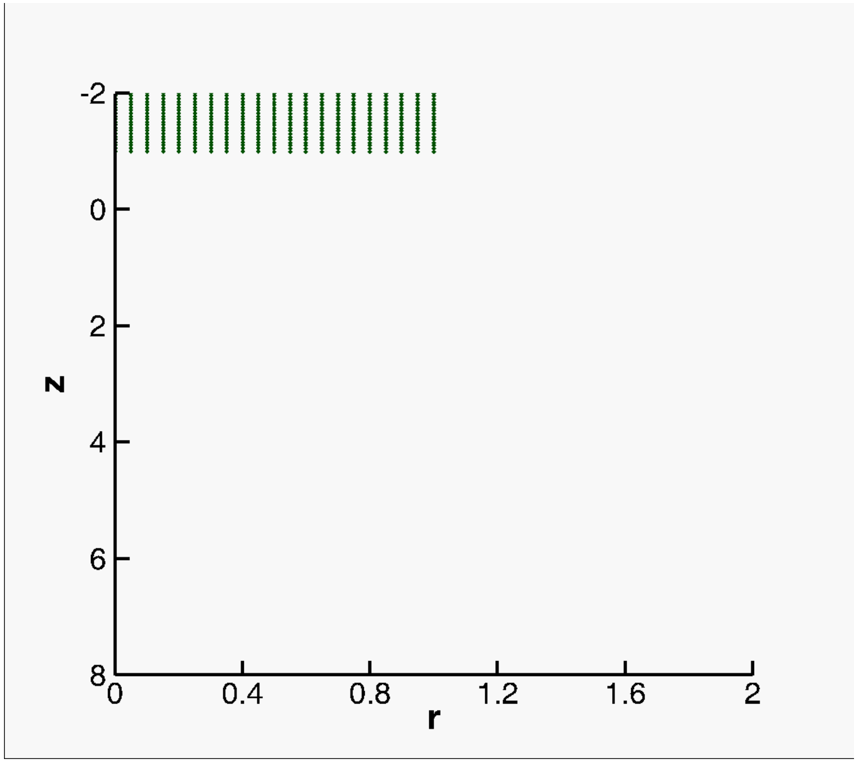


Droplets: numerical simulation

A dust cloud behind the vortex ring, Re = 100

$$\sigma = 10\mu m$$

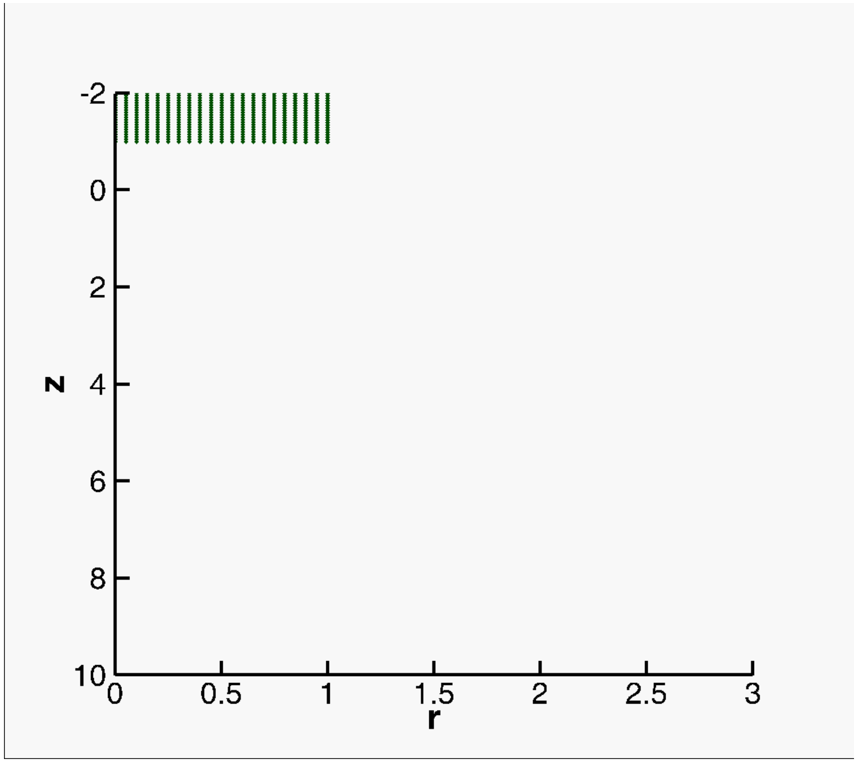




Droplets: numerical simulation

A dust cloud behind the vortex ring, Re = 100

$$\sigma = 10 \mu m$$



Conclusions:

- Unsteady axially symmetric two phase vortex ring flow modelled using FLA and Kaplanski analytical solution
- Further investigation to compare numerical results with the experimental data is needed