The Sir Harry Ricardo Laboratories

Research Workshop New approaches to spray modelling

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Modelling of non-isothermal sprays using a combined viscous vortex method and the Fully Lagrangian Approach

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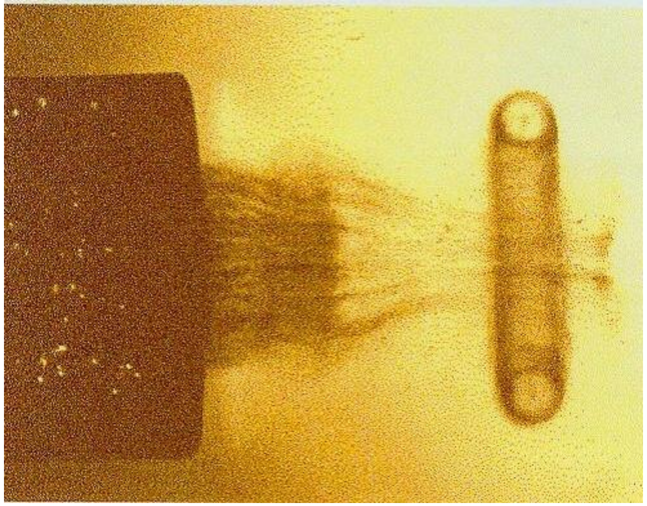
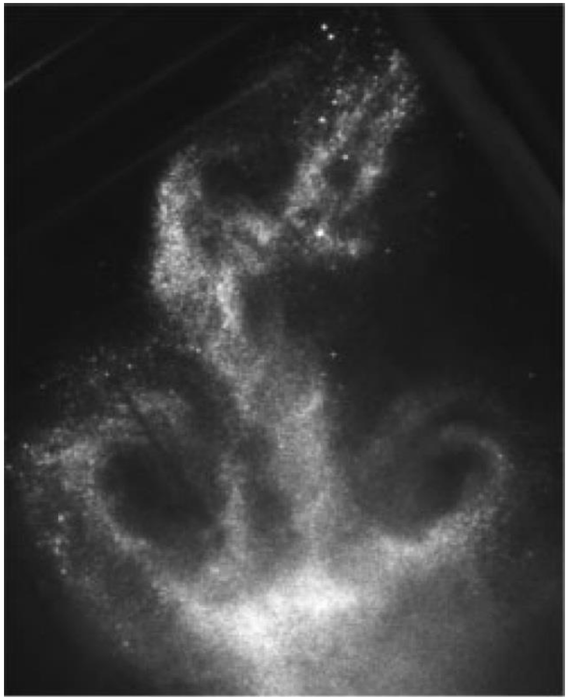
- Numerical algorithm
- Two-phase jet
- Conclusion

Introduction



Formation of a Vortex Ring exiting a gun muzzle as captured by Dr. Lyons with spark photography upon firing a blank 40mm grenade (1997)



A typical high-speed photograph of a G-DI spray (Begg et al 2009)

EPSRC project ‘Development of the full Lagrangian approach for the analysis of vortex ring-like structures in disperse media: application to gasoline engines’

Problem formulation

- 2D transient flow
- One-way coupled two-fluid approach
  - Carrier phase: incompressible viscous fluid (droplet liquid vapour)
  - Dispersed phase: cloud of identical evaporating droplets, $\dot{m} = q_s / H$
- Force acting on a single particle:

$$\mathbf{f}_s = 6\pi\sigma^* \mu \left(\mathbf{v}^* - \mathbf{v}_s^* \right) \left(1 + \frac{1}{6} \mathrm{Re}_s^{2/3} \right)$$

- Heat rate:

$$q_s = 4\pi\sigma^* \lambda \left(T^* - T_s^* \right) \left(1 + 0.3 \,\mathrm{Pr}^{1/3} \,\mathrm{Re}_s^{1/2} \right)$$

Non-dimensional parameters

$$\mathbf{r}\left(\mathbf{r}_s\right) = \frac{\mathbf{r}^*\left(\mathbf{r}_s^*\right)}{L},\ \mathbf{v}\left(\mathbf{v}_s\right) = \frac{\mathbf{v}^*\left(\mathbf{v}_s^*\right)}{U},$$

$$T\left(T_s\right) = \frac{T_\infty - T^*\left(T_s^*\right)}{T_\infty - T_i},\ \sigma = \frac{\sigma^*}{\sigma_s},$$

$$\mathbf{r} = \left(x, y\right),\ \mathbf{r}_s = \left(x_s, y_s\right),\ \mathbf{v} = \left(u, v\right),\ \mathbf{v}_s = \left(u_s, v_s\right)$$

6 similarity parameters:

Re, γ, Pr, β, δ, Re_{s0}

$$\beta = \frac{6\pi\sigma_s\mu L}{m_s U},\ \delta = \frac{8\pi}{3}\frac{\sigma_s L\lambda\left(T_\infty - T_i\right)}{m_s U H},$$

$$\mathrm{Re}_s = \mathrm{Re}_{s0}\sigma\left|\mathbf{v} - \mathbf{v}_s\right|,\ \mathrm{Re}_{s0} = \frac{2\rho\sigma_s U}{\mu}.$$

Problem formulation

$$\frac{\partial \omega}{\partial t} + \mathrm{div}\left(\omega \mathbf{v}\right) = \frac{1}{\mathrm{Re}} \Delta \omega$$

$$\frac{\partial T}{\partial t} + \mathrm{div}(T\mathbf{v}) = \frac{\gamma}{\mathrm{Re}\,\mathrm{Pr}} \Delta T$$

$$\frac{d\mathbf{r}_s}{dt} = \mathbf{v}_s,$$

$$\frac{d\mathbf{v}_s}{dt} = \frac{\beta}{\sigma^2}\left(\mathbf{v} - \mathbf{v}_s\right)\Psi_d,$$

$$\frac{d\sigma^2}{dt} = \delta\left(T - T_s\right)\Psi_h,$$

$$\Psi_d = 1 + \frac{1}{6}\mathrm{Re}_s^{2/3}, \;\; \Psi_h = 1 + 0.3\mathrm{Pr}^{1/3}\mathrm{Re}_s^{1/2}$$

Initial conditions

$$\omega\left(\mathbf{r}, t_0\right) = \omega_0\left(\mathbf{r}\right)$$

$$T\left(\mathbf{r}, t_0\right) = T_0\left(\mathbf{r}\right)$$

$$\mathbf{r}_s = \mathbf{r}_0, \;\; \mathbf{v}_s = \mathbf{v}_{s0}$$

$$T_s = T_{s0}\left(x, y\right), \;\; \sigma = \sigma_0\left(x, y\right)$$

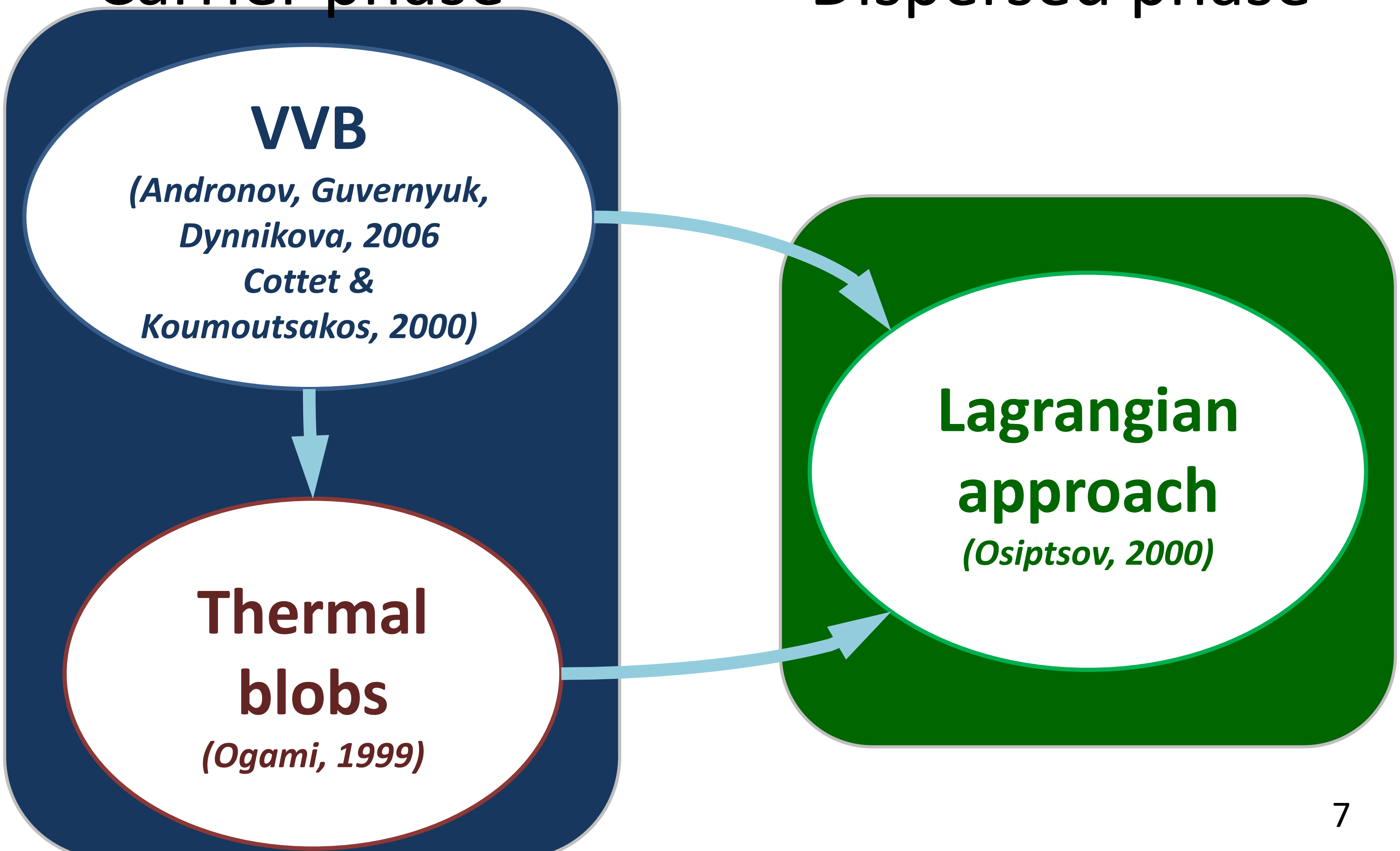
Boundary conditions

$$\mathbf{r} \to \infty : \;\; \omega \to 0, \;\; T \to 0$$

Numerical algorithm

Carrier phase

Dispersed phase



VVM

$$\frac{\partial\omega}{\partial t} + \operatorname{div}(\omega\mathbf{v}) = \frac{1}{\mathrm{Re}}\Delta\omega$$

$$\frac{\partial\omega}{\partial t} + \operatorname{div}\left(\omega\left(\mathbf{v} + \mathbf{v}_{dv}\right)\right) = 0$$

$$\mathbf{v}_{dv} = -\frac{1}{\mathrm{Re}}\frac{\nabla\omega}{\omega}$$

$$\frac{d\mathbf{r}_{vi}}{dt} = \mathbf{v}\left(\mathbf{r}_{vi}, t\right) + \mathbf{v}_{dv}\left(\mathbf{r}_{vi}, t\right)$$

TBM

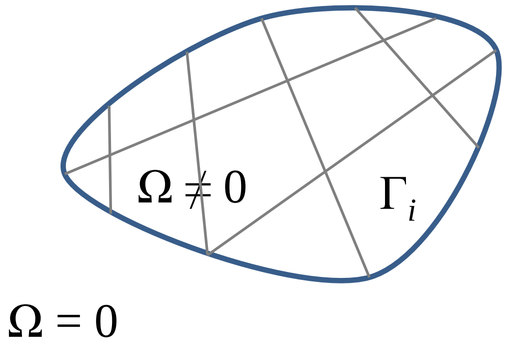
$$\frac{\partial T}{\partial t} + \operatorname{div}(T\mathbf{v}) = \frac{\gamma}{\mathrm{Re}\,\mathrm{Pr}}\Delta T$$

$$\frac{\partial T}{\partial t} + \operatorname{div}\left(T\left(\mathbf{v} + \mathbf{v}_{dT}\right)\right) = 0$$

$$\mathbf{v}_{dT} = -\frac{\gamma}{\mathrm{Re}\,\mathrm{Pr}}\frac{\nabla T}{T}$$

$$\frac{d\mathbf{r}_{Ti}}{dt} = \mathbf{v}\left(\mathbf{r}_{Ti}, t\right) + \mathbf{v}_{dT}\left(\mathbf{r}_{Ti}, t\right)$$

Numerical algorithm



$$\Gamma_i = \omega_0\left(\mathbf{r}_{vi}\left(t_0\right)\right)\Delta_{vi} = \text{const}$$

$$\omega\left(\mathbf{r}, t\right) \approx \sum_{i=1}^{N} \Gamma_i \zeta_{\varepsilon_i}\left(\mathbf{r} - \mathbf{r}_{vi}\left(t\right)\right)$$

$$\Theta_i = T_0\left(\mathbf{r}_{Ti}\left(t_0\right)\right)\Delta_{Ti} = \text{const}$$

$$T\left(\mathbf{r}, t\right) \approx \sum_{i=1}^{M} \Theta_i \zeta_{\varepsilon_i}\left(\mathbf{r} - \mathbf{r}_{Ti}\left(t\right)\right)$$

$$\mathbf{v}_{dv} = -\frac{1}{\mathrm{Re}} \frac{\sum_{i=1}^{N} \Gamma_i \cdot \nabla \zeta_{\varepsilon_i}\left(\mathbf{r} - \mathbf{r}_{vi}\left(t\right)\right)}{\sum_{i=1}^{N} \Gamma_i \zeta_{\varepsilon_i}\left(\mathbf{r} - \mathbf{r}_{vi}\left(t\right)\right)} \qquad \mathbf{v}_{dT} = -\frac{\gamma}{\mathrm{Re}\,\mathrm{Pr}} \frac{\sum_{i=1}^{M} \Theta_i \cdot \nabla \zeta_{\varepsilon_i}\left(\mathbf{r} - \mathbf{r}_{Ti}\left(t\right)\right)}{\sum_{i=1}^{M} \Theta_i \zeta_{\varepsilon_i}\left(\mathbf{r} - \mathbf{r}_{Ti}\left(t\right)\right)}$$

$$\zeta_{\varepsilon_i}\left(\mathbf{r}\right) = \frac{1}{3\pi\varepsilon_i^2}\left(4 - \frac{\mathbf{r}^2}{\varepsilon_i^2}\right)\exp\left(-\frac{\mathbf{r}^2}{\varepsilon_i^2}\right)$$

Numerical algorithm

Biot-Savart law and the mollified kernel:

$$\mathbf{v}(\mathbf{r}) = \frac{1}{2\pi}\sum_{i=1}^{N}\Gamma_i \mathbf{e}_z \times \frac{\mathbf{r}-\mathbf{r}_{vi}}{\left|\mathbf{r}-\mathbf{r}_{vi}\right|^2} \approx$$

$$\approx \frac{1}{2\pi}\sum_{i=1}^{N}\Gamma_i \mathbf{e}_z \times \mathbf{F}_{\varepsilon_i}(\mathbf{r}-\mathbf{r}_{vi}),$$

$$\mathbf{F}_{\varepsilon_i}(\mathbf{r}) = \frac{\mathbf{r}}{2\pi\mathbf{r}^2}\left[1+\left(\frac{\mathbf{r}^2}{\varepsilon_i^2}-1\right)\exp\left(-\frac{\mathbf{r}^2}{\varepsilon_i^2}\right)\right]$$

Two-phase jet injection

$$\mathrm{Re} = 1000,\ \mathrm{Pr} = 0.8,\ \gamma = 1.33$$

σ^*, μm	β	δ
100	0.05	0.0014
50	0.21	0.006
10	5.5	0.14

Initial conditions:

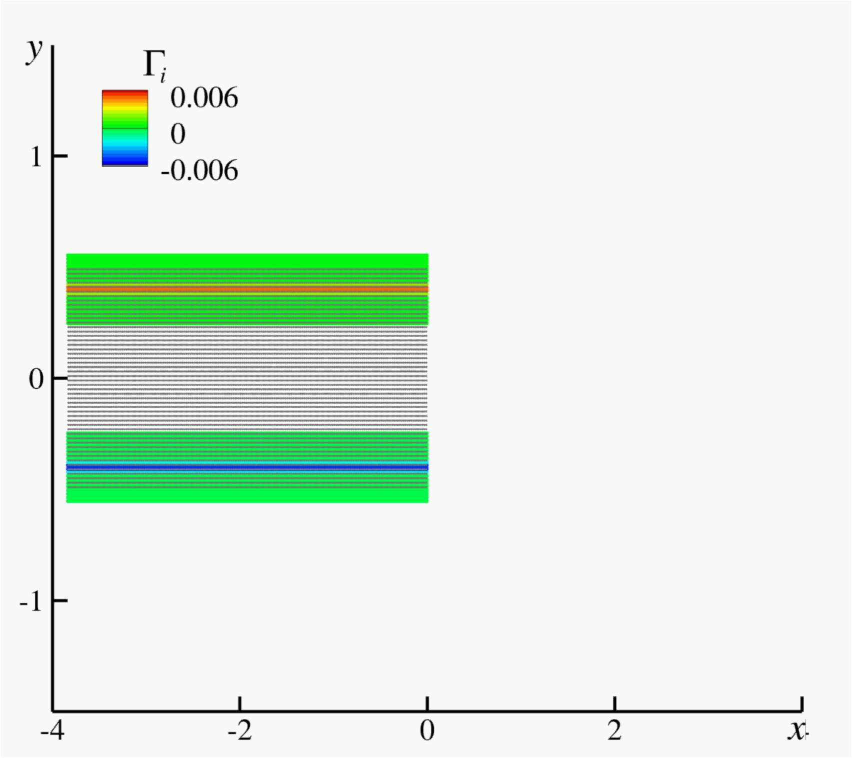
$$\begin{cases} u_0 = \dfrac{1}{1+\exp\left(-100\left(y+0.4\right)\right)} + \dfrac{1}{1+\exp\left(100\left(y-0.4\right)\right)} - 1, \\ v_0 = 0. \end{cases}$$

$$T_0\left(x,y\right) = u_0\left(x,y\right)$$

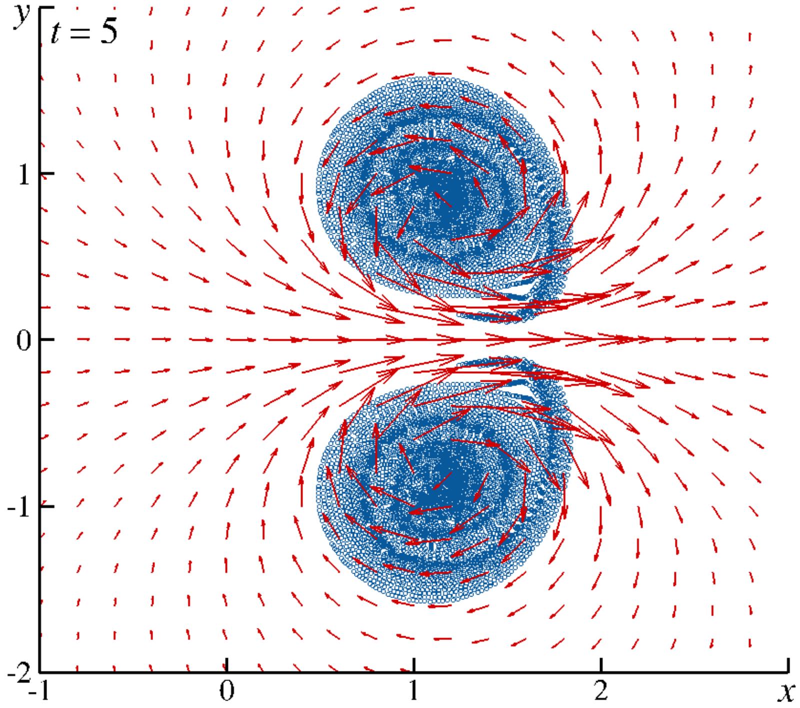
$$\mathbf{r}_s = \mathbf{r}_0,\ \mathbf{v}_s = \left(u_{s0}, v_{s0}\right) = \left(0.8, 0\right)$$

$$T_s\left(x,y\right) = 1,\ \sigma = 1$$

Two-phase jet injection

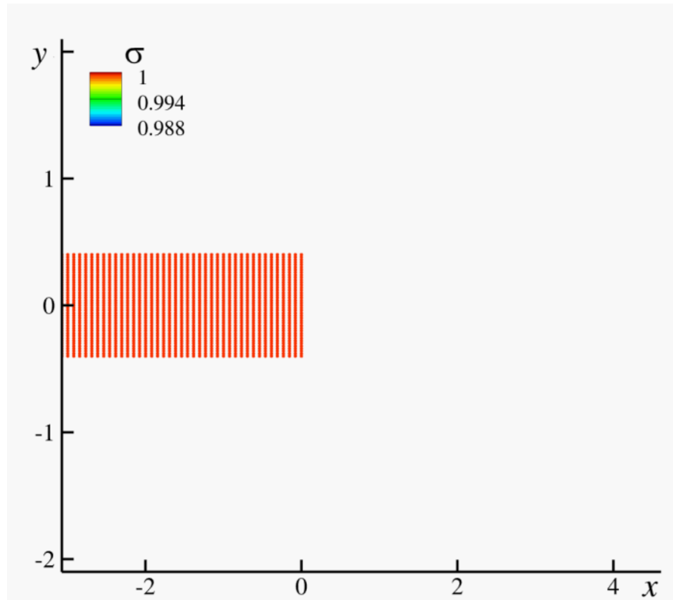


Motion of viscous-vortex and thermal blobs



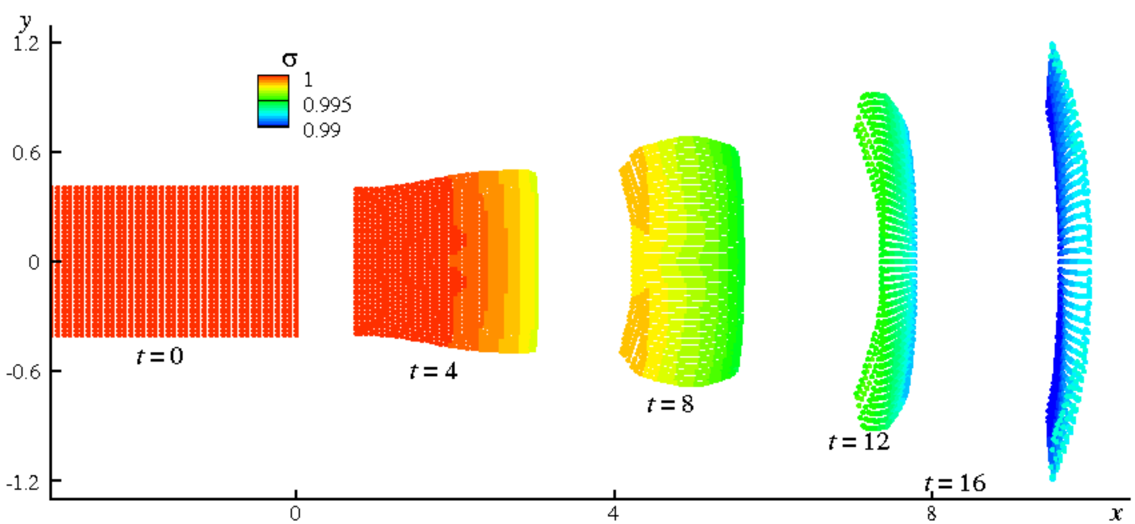
Viscous-vortex blobs and velocity field

Two-phase jet injection

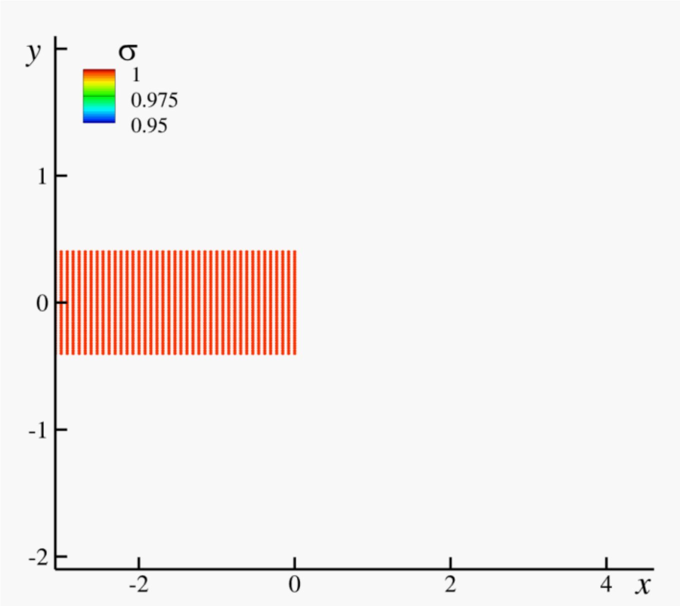


Droplet distributions and their sizes

$\beta = 0.05, \ \delta = 0.0014$

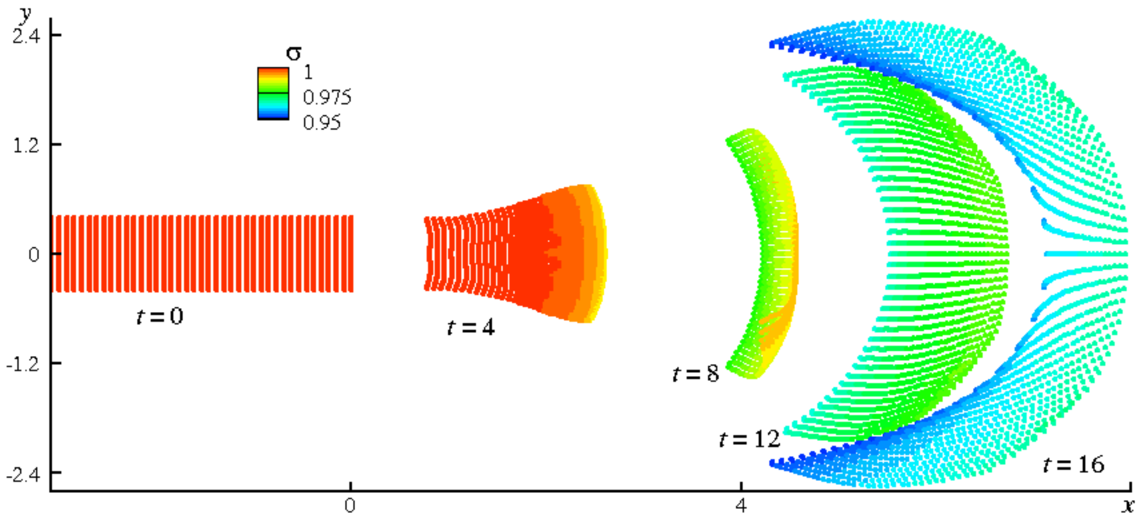


Two-phase jet injection

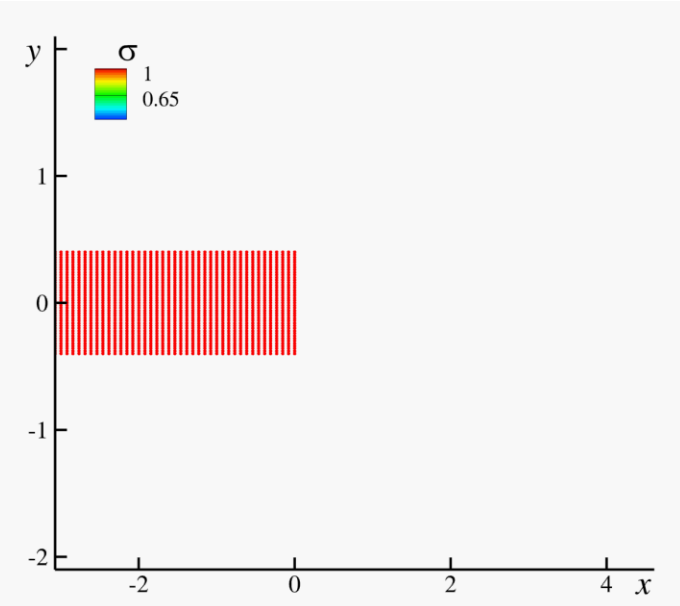


Droplet distributions and their sizes

$\beta = 0.21,\ \delta = 0.006$

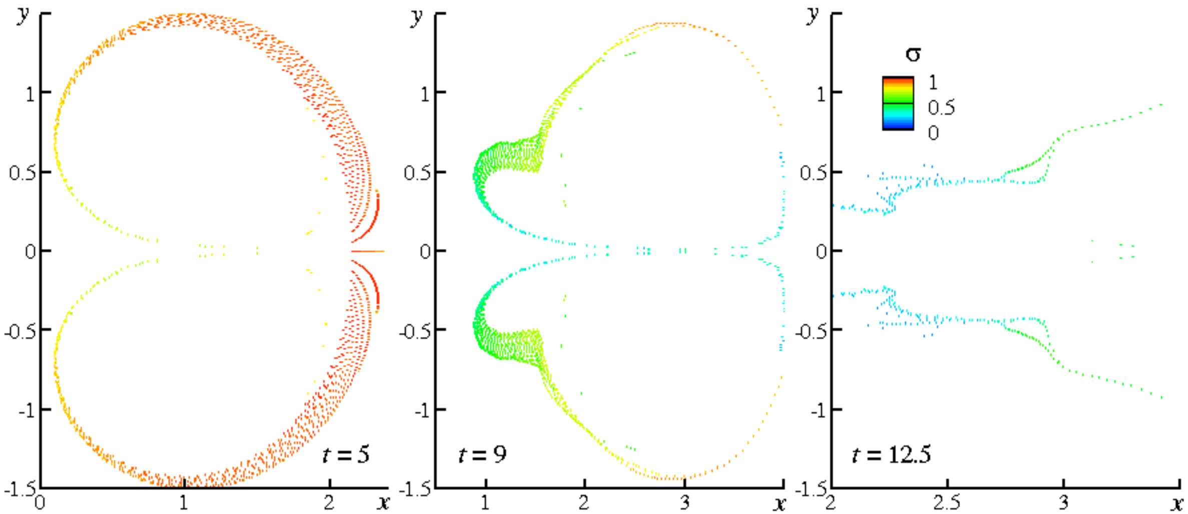


Two-phase jet injection



Droplet distributions and their sizes

$\beta = 5.5, \;\; \delta = 0.14$



Conclusion

- A meshless method based on a combination of the viscous-vortex and thermal-blob methods for the carrier phase and the Lagrangian approach for the dispersed phase is developed. It can be applied for modelling unsteady 2D flows of ‘gas-evaporating droplets’ systems
- The numerical code was verified for the Lamb vortex flow.
- The injection of a cold ‘vapour – droplet’ jet into a hot quiescent gas was studied. Three cases corresponding to three initial droplet diameters were considered. The flow with the smallest droplets showed better mixing among the three cases considered: these droplets formed ring-like structures. The medium size droplets collected into narrow bands, the droplets at the band ends were shown to slow down and evaporate more rapidly than at the centre. The cloud of droplets with the greatest inertia remained close to the jet axis. In the three cases considered, the evaporation process led to various droplet relative size distributions: (i) in the case of small droplets, larger droplets collected behind the vortex pair; (ii) in the case of medium size droplets, larger droplets remained at the front of the two-phase jet; (iii) in the case of the largest droplets, larger droplets remained at the rear part of the two-phase region, though the size varied negligibly with time.