21th August, 2017
Advanced Engineering Centre (AEC)

Modelling of confined swirling vortex rings

Andreas Papoutsakis.

University of Brighton

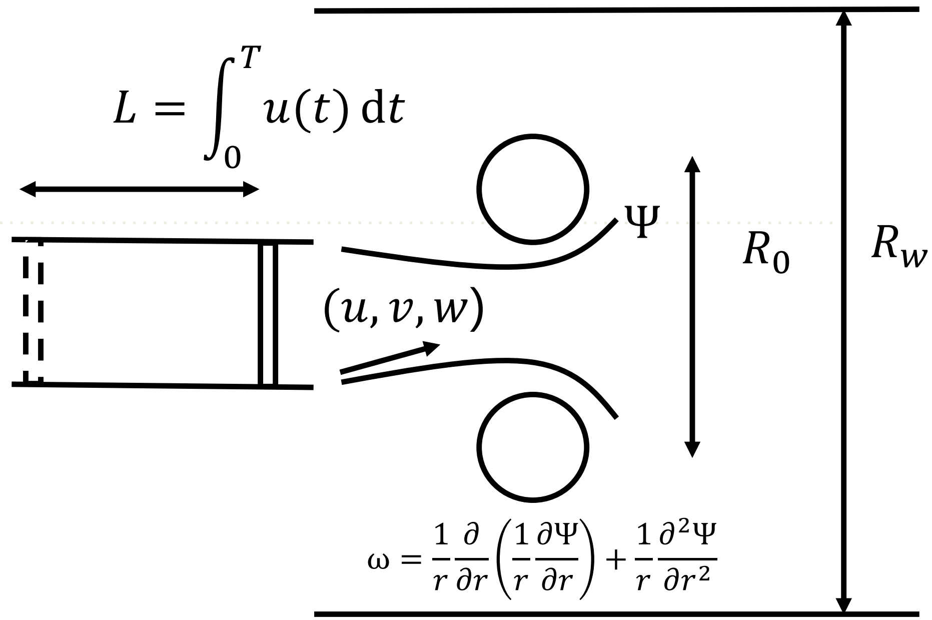
Presentation Outline

1. Confined Swirling Vortex Ring Topology
2. Technical Interest/Applications
3. Energy conservation equation
4. VR flow solution with swirl
5. Confinement modelling
6. JetLES
7. Results
8. Closure: Conclusions / Questions.

Vortex Ring Topology

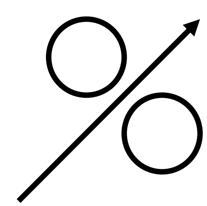
VR characteristics

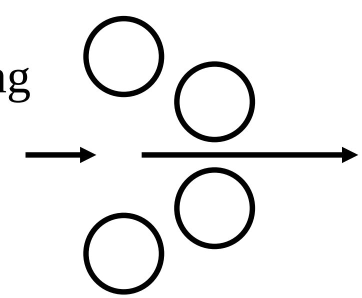
1. Created by injection in ambient fluid
2. The length of the stroke L dictates the characteristics of the resulting VR
3. Modelling of the VR is based on a vorticity/stream-function representation of the flow field



$$u = -\frac{1}{r}\frac{\partial \Psi}{\partial r}, \qquad v = \frac{1}{r}\frac{\partial \Psi}{\partial x}, \quad w = u_\phi$$

Alternative configurations
Hollow cone / Leap frogging

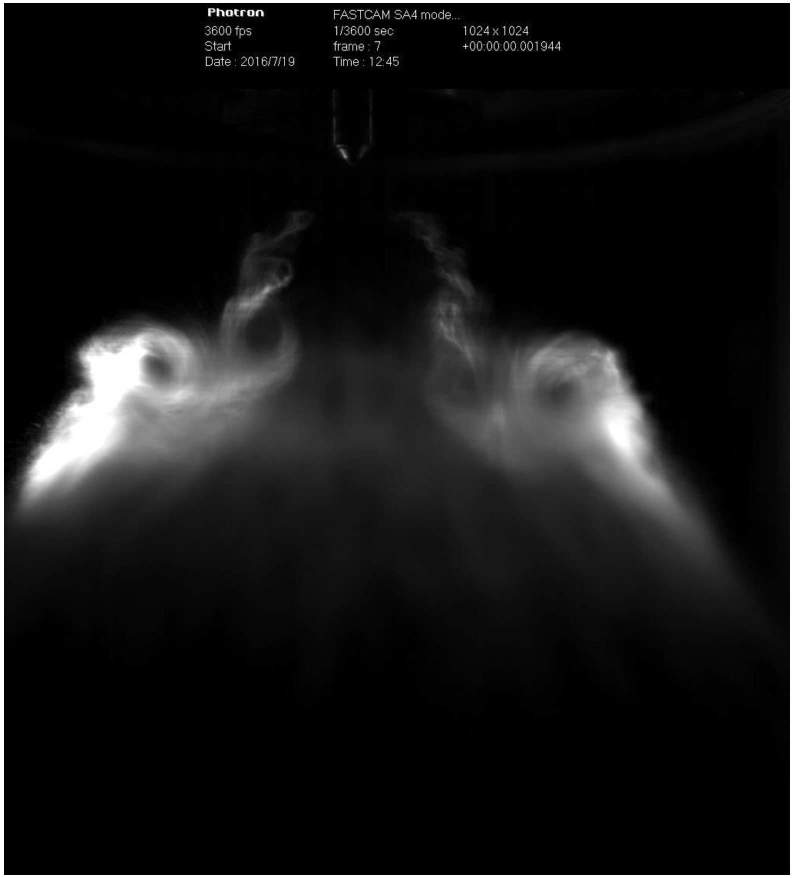




Technical Interest/Applications

Vortex Rings

Begg, S. *et al.* 2016



1. Injection in IC engines.
2. Aircraft/Bird flight (helicopters, hummingbirds).
3. Swimming organisms (octopus siphon, tuna wake) and swimming devices.
4. Environmental Flows (VR cannon chimneys, volcanos).

Technical Interest/Applications

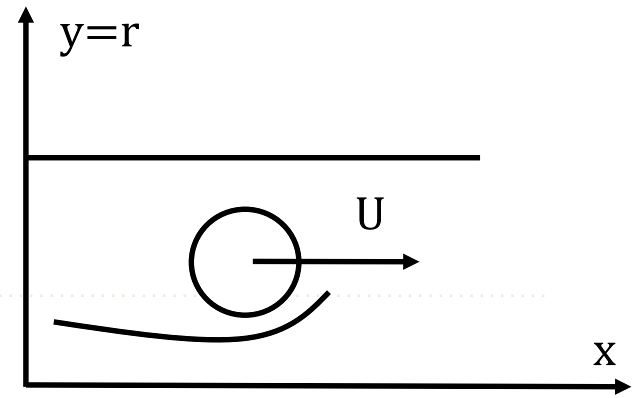
Swirl

Athanasiadis, A. and Hart, D. 2016

1. Swirling injectors.
2. As an instability mode in non-swirling VR.
3. Roll control in swimming devices?
4. Controlling the VR evolution and positioning in injection systems?

VR is a self-contained vortical structure that can travel long distances. Although VR appear in a variety of applications, very few applications of their unique characteristics are exploited.

Vortex Ring Modelling (Translation)



1. The modelling of the **U** of the VR is based upon the conservation of the translational KE of the VR and the KE of the VR (rotation + swirl). Valid for a formed VR in un-bounded V.

$$E = \frac{1}{2}\int_V \boldsymbol{u}^2 dV = \int_V \boldsymbol{u}\cdot(\boldsymbol{x}\times\omega)dV + \frac{1}{2}\int_S (\boldsymbol{u}\cdot\boldsymbol{x})(\boldsymbol{u}\cdot\boldsymbol{n})dS - \frac{1}{2}\int_S \boldsymbol{u}^2(\boldsymbol{n}\cdot\boldsymbol{x})dS$$

Safmann 3.11.3

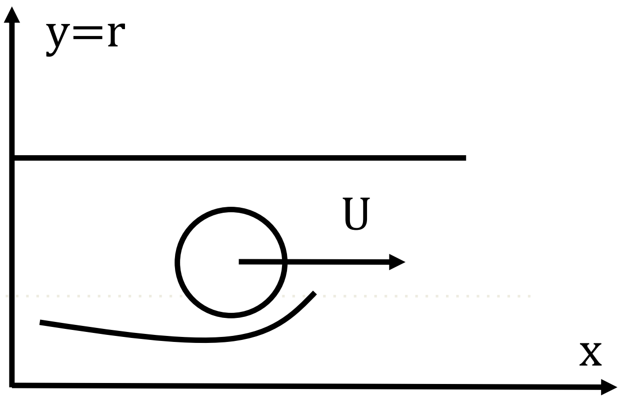
$$\boldsymbol{u} = \hat{\boldsymbol{u}} + \mathbf{U} = \nabla\times\boldsymbol{\Psi} + w\cdot\mathbf{z}$$

$$E = 2U\cdot I + \int_V \hat{\boldsymbol{u}}\cdot(\boldsymbol{x}\times\omega)dV \text{ where } I = \frac{1}{2}\int_V \boldsymbol{x}\times\boldsymbol{\omega}\, dV$$

E of mean E of VR Shafmann 10.3.2

Vortex Ring Modelling (Translation)

$$E = 2\boldsymbol{U}\cdot\boldsymbol{I} + \int_V \hat{\boldsymbol{u}}\cdot(\boldsymbol{x}\times\boldsymbol{\omega})dV$$ Safmann §10.3



Impulse in cyl. Coordinates:

$$I = \frac{1}{2}\int_V \boldsymbol{x}\times\boldsymbol{\omega}\, dV = \pi\int_A y^2\omega dA$$

E in cyl. Coordinates ($yu^2 + yv^2 = \omega_\varphi\psi$) Safmann §3.11.7

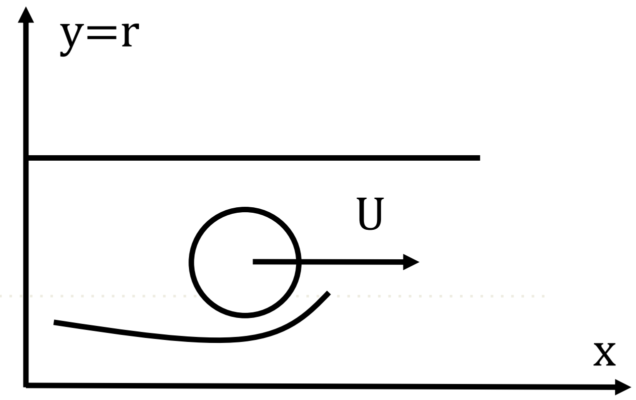
$$E = \frac{1}{2}\int_V \boldsymbol{u}^2 dV \overset{dV=2\pi y dA}{=} \pi\int_A \omega_\varphi\psi dA + \pi\int_A yw^2\, dA$$

VR Energy $\int_V \hat{\boldsymbol{u}}\cdot(\boldsymbol{x}\times\boldsymbol{\omega})dV$ in cyl. Coordinates

$$\int_V \hat{\boldsymbol{u}}\cdot(\boldsymbol{x}\times\boldsymbol{\omega})dV = \pi\int_A yw^2 dA + 2\pi\int_A y\omega_\varphi(yu - xv)dA$$

Vortex Ring Modelling (Translation)

$$\int_V \hat{\boldsymbol{u}} \cdot (\boldsymbol{x} \times \boldsymbol{\omega}) dV$$



Lamp's transformation u,v →w:

$$2\pi \int_A y\omega_\varphi (yu - xv) dA = -6\pi \int_A xyv\omega_\varphi dA + 2\pi \int_A yw^2\, dA$$

Energy separation

$$U = \frac{E - \int_V \hat{\boldsymbol{u}} \cdot (\boldsymbol{x} \times \boldsymbol{\omega}) dV}{2I}$$

$$U = \frac{\int_A \omega_\varphi \psi dA + 6 \int_A xyv\omega_\varphi dA - 2 \int_A yw^2\, dA}{2\pi \int_A y^2 \omega dA}$$

Vortex Ring Flow Field.

$$U = \frac{\int_A \omega_\varphi \psi dA + 6 \int_A xyv\omega_\varphi dA - 2 \int_A yw^2 \, dA}{2\,\pi \int_A y^2 \omega dA}$$

Knowing the u, v, w of the VR we obtain U, M and J

1. Thin vorticity filament solution for a VR (Saffman)
2. Thick viscous solution for a VR (Kaplanski F., Fukumoto Y.)

uv momentum M (vorticity momentum) and swirl momentum J conserve in time for a formed VR

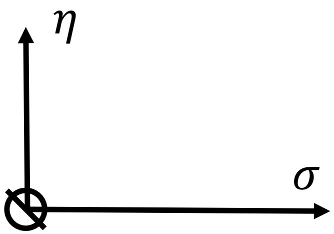
$$M = \pi \int_{\mathrm{A}} \omega \, y^2 dA \qquad J = 2\,\pi \int_{\mathrm{A}} u_\varphi \, y^2 dA$$

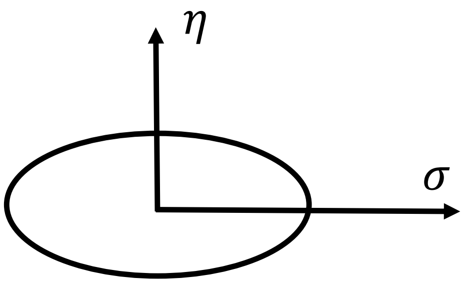
Vortex Ring Flow Field

$$\sigma = \frac{x}{R_0\sqrt{2t}} \qquad \eta = \frac{y}{R_0\sqrt{2t}} \qquad \theta = \frac{R_0}{R_0\sqrt{2t}}$$

Thick VR assumption, Independent solutions for the VR vorticity and the VR swirl velocity.

Time t=0 $\omega_0 = w_0 = \delta(x_1)\delta(r_1 - 1)$





At time t:

$$\omega = \omega_0\, e^{\left(-\frac{\sigma^2+\eta^2+\theta^2}{2}\right)} \mathrm{I}_1(\sigma\theta) \qquad w = w_0\, e^{\left(-\frac{\sigma^2+\eta^2+\theta^2}{2}\right)} \mathrm{I}_1(\sigma\theta)$$

Confined Swirling Vortex Ring Flow Field

$$\sigma = \frac{x}{R_0\sqrt{2t}} \qquad \eta = \frac{y}{R_0\sqrt{2t}} \qquad \theta = \frac{R_0}{R_0\sqrt{2t}}$$

The Stream-Function and the confinement correction $\Psi = \Psi_V - \Psi_0$ for a confined VR are:

$$\Psi_V = \frac{\eta\theta}{4}\int_0^\infty F(\eta,\sigma,\theta)J_1(\eta\theta\mu)J_1(\theta\mu)d\mu$$

$$\Psi_0 = \frac{\eta}{\pi}\int_0^\infty \frac{K_1(\mu/\varepsilon)}{I_1(\mu/\varepsilon)}\mathrm{I}_1(\eta\mu)\mathrm{I}_1(\mu)\cos(\mu\sigma)\,d\mu$$

At time t:

$$u = -\frac{1}{r}\frac{\partial\Psi}{\partial r}, \qquad v = \frac{1}{r}\frac{\partial\Psi}{\partial x}, \qquad w = w_0\, e^{\left(-\frac{\sigma^2+\eta^2+\theta^2}{2}\right)}\mathrm{I}_1(\sigma\theta)$$

Closure of U, Γ, E.

$$\sigma = \frac{x}{R_0\sqrt{2t}} \qquad \eta = \frac{y}{R_0\sqrt{2t}} \qquad \theta = \frac{R_0}{R_0\sqrt{2t}}$$

Following Kaplanski the closed formulation for U is:

$$U = \frac{M\theta\sqrt{\pi}}{4\pi^2 R_0^3}\{3e^{-\frac{\theta^2}{2}} I_1\left(\frac{\theta^2}{2}\right) + \frac{\theta^2}{12^2} F_2\left(\left\{\frac{3}{2},\frac{3}{2}\right\},\left\{\frac{5}{2},3\right\},-\theta^2\right)$$
$$-\frac{3\theta^2}{5^2} F_2\left(\left\{\frac{3}{2},\frac{5}{2}\right\},\left\{\frac{7}{2},\frac{5}{2}\right\},-\theta^2\right) - 2\,\Omega_1\theta^2 e^{-\frac{\theta_1^2}{2}} I_1\left(\frac{\theta_1^2}{2}\right)\}$$

From the thick VR solution the circulation and the energy of the VR can also be closed as:

$$\Gamma = \pi\int_A \omega\, dA \qquad E = \frac{1}{2}\int_V \boldsymbol{u}^2 dV = \pi\int_A \omega_\varphi \psi\, dA + \pi\int_A y w^2\, dA$$

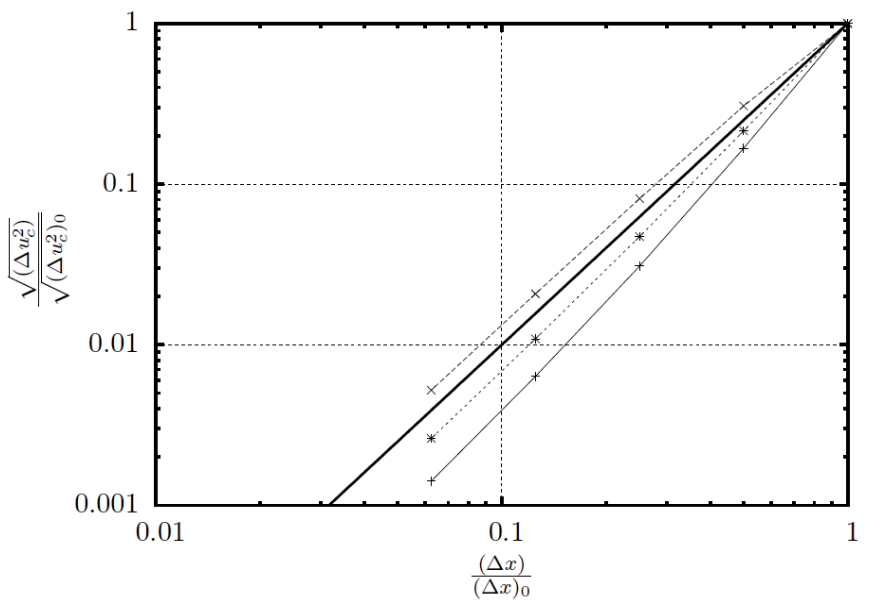
JETLes

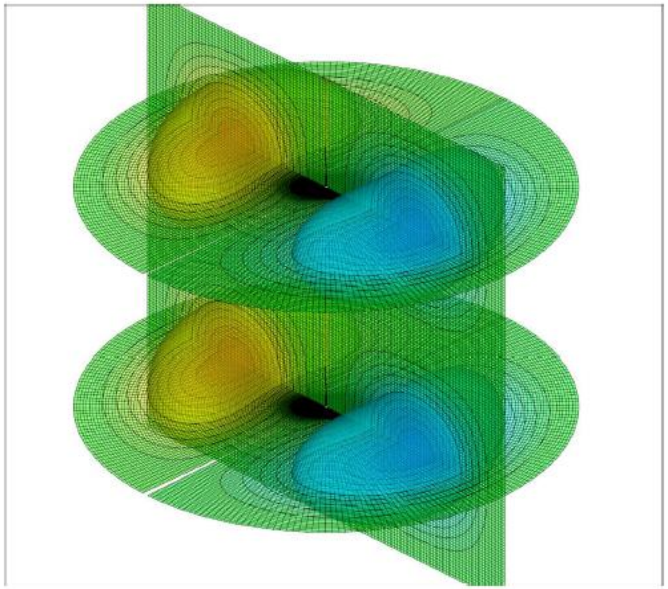
Incompressible solver with a spectral/physical space Poisson solver (Danaila, I. Luddens, F.)

Scalable TDMA solvers for the Poisson and momentum Equations.

Second order accurate

$$u_\theta = (1 - \cos \pi r)(1 - \cos \pi z) \; ; u_r = 0 \; ; u_z = 0 \; .$$



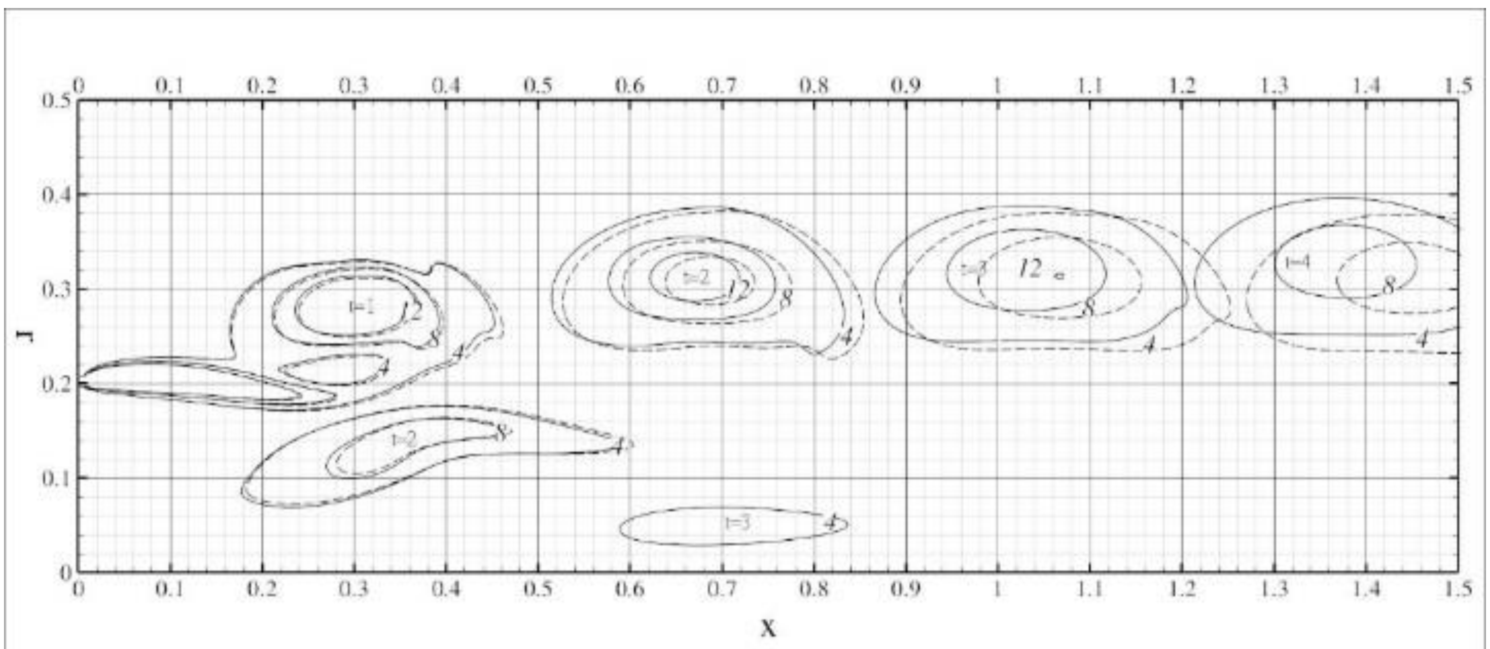


Results

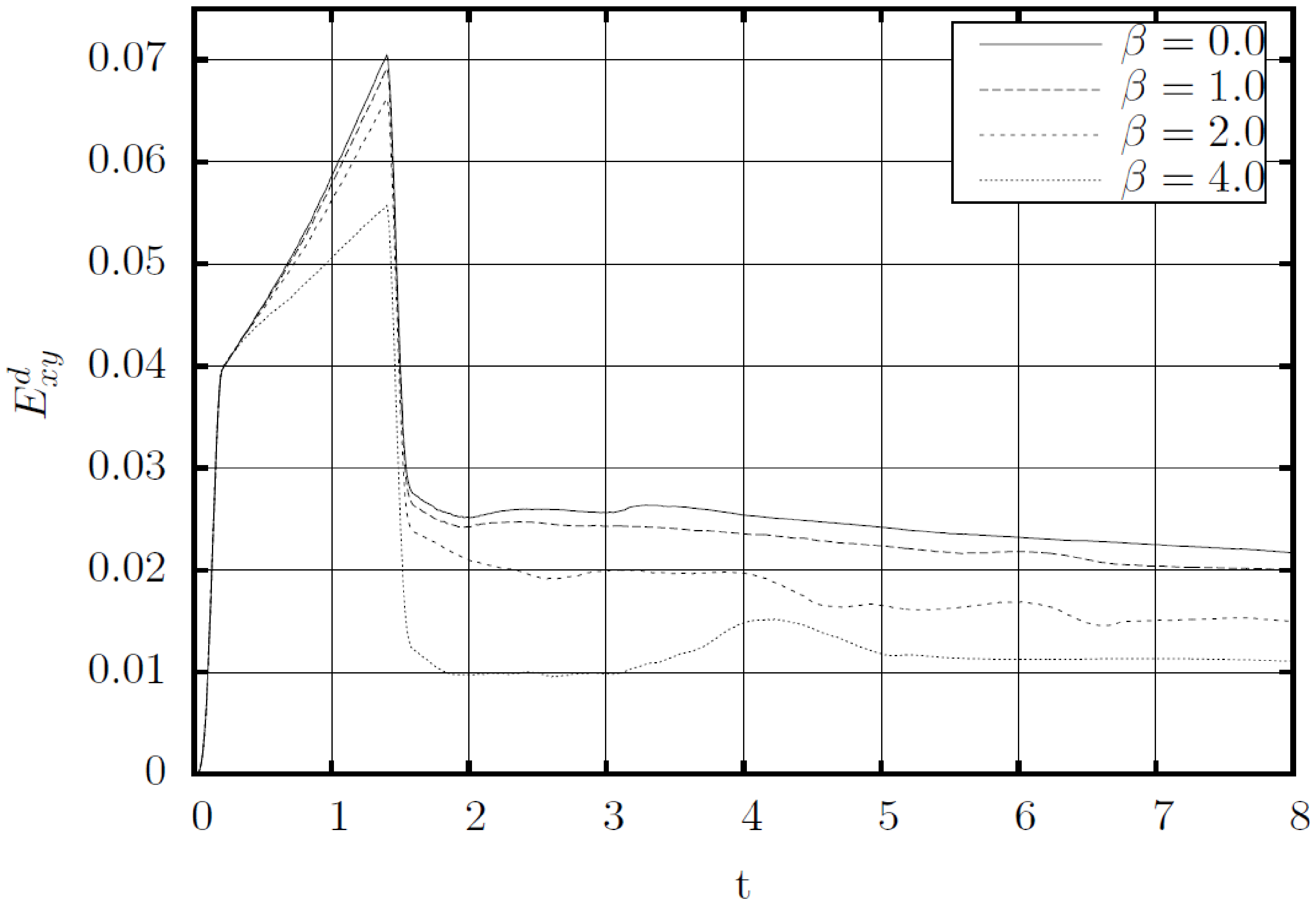
Simulations for For Re_D=1000, and β=0, 0.2, 0.4, 0.8 L/D=3.5

$$u_x(r,t)/u_0 = \begin{cases} 3\left(\frac{t}{t_1}\right)^2 - 2\left(\frac{t}{t_1}\right)^3, & t < t_1 \\ 1, & t_1 < t < T - t_1 \\ 3\left(\frac{T-t}{t_1}\right)^2 + 2\left(\frac{T-t}{t_1}\right)^3, & t > T - t_1 \end{cases},$$

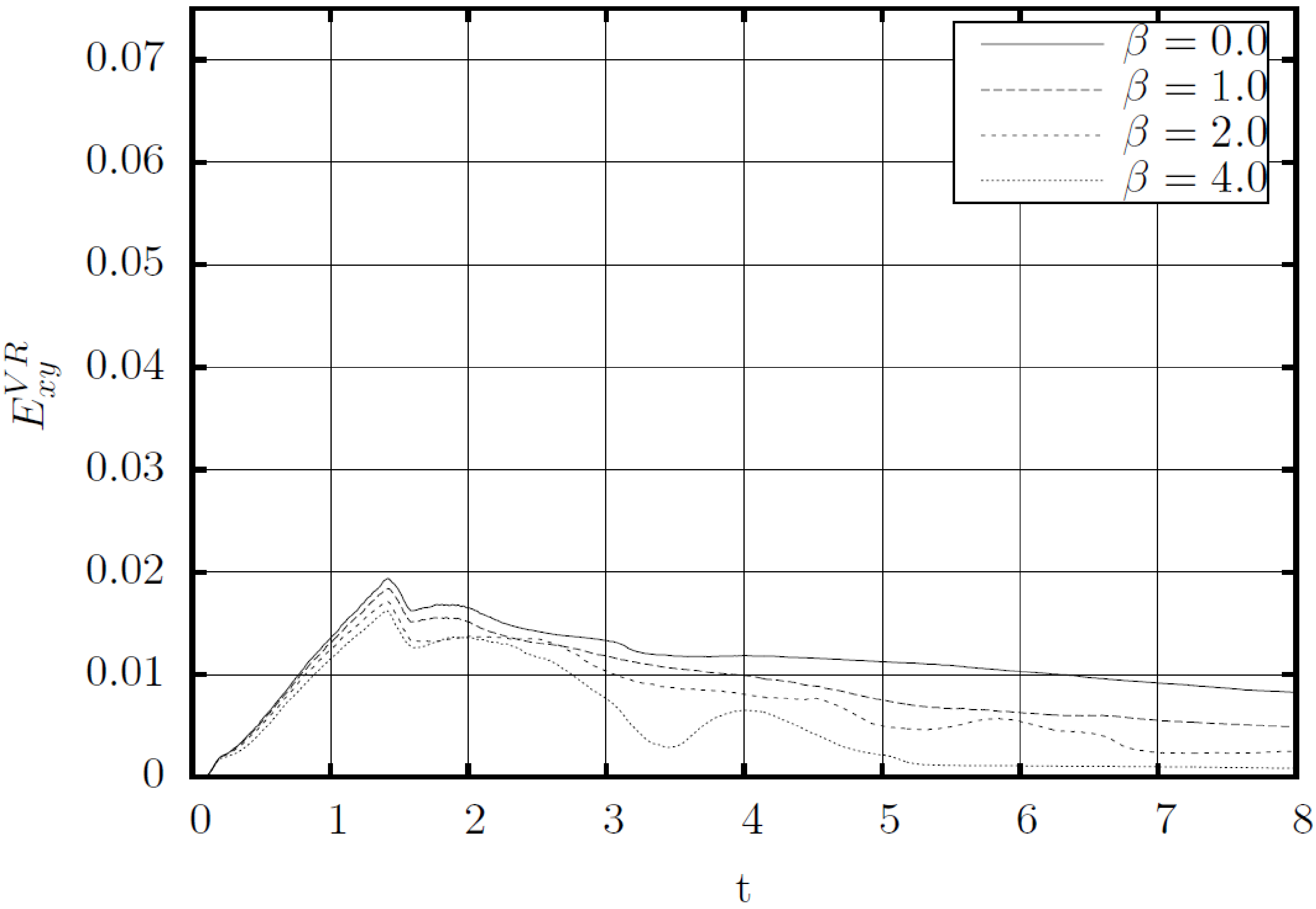
$$u_\varphi = \beta \frac{r}{D} u_x(r,t)$$



Results

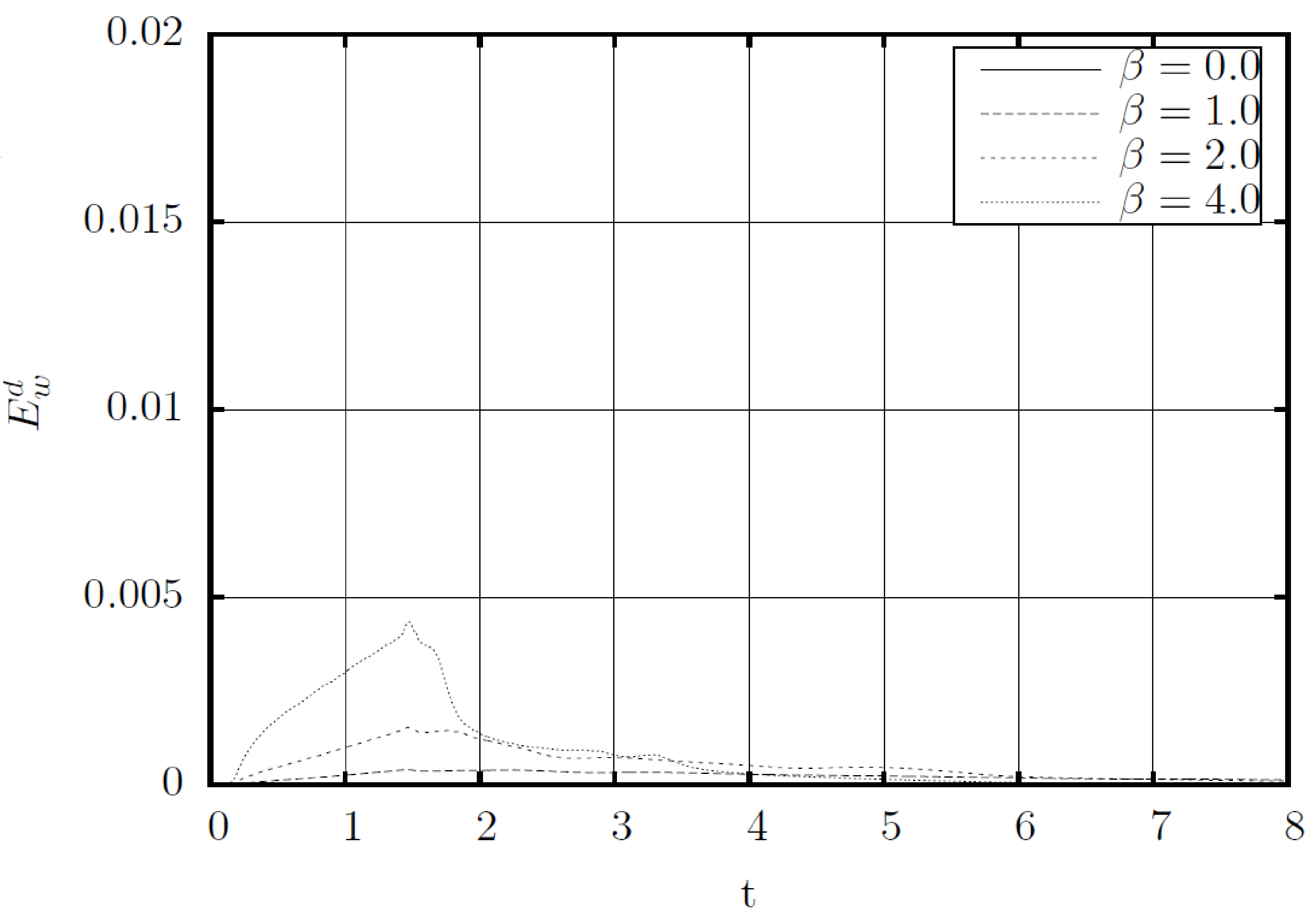

$\beta = 0.0$
$\beta = 1.0$
$\beta = 2.0$
$\beta = 4.0$
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
E^d_{xy}
0
1
2
3
4
5
6
7
8
t

Results

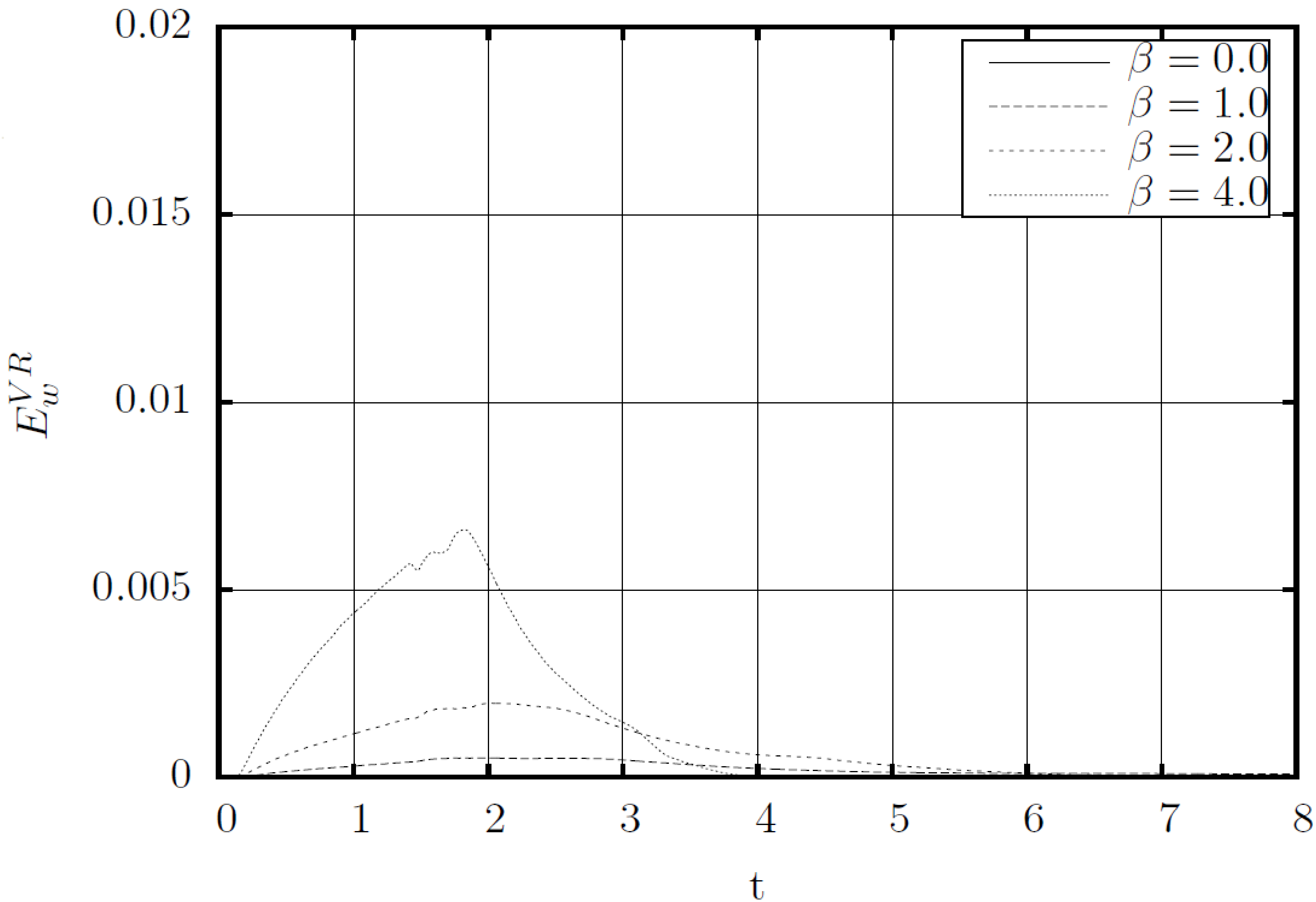


$\beta = 0.0$
$\beta = 1.0$
$\beta = 2.0$
$\beta = 4.0$
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
E_{xy}^{VR}
0
1
2
3
4
5
6
7
8
t

Results


$\beta = 0.0$
$\beta = 1.0$
$\beta = 2.0$
$\beta = 4.0$
E_w^d
0.02
0.015
0.01
0.005
0
0
1
2
3
4
5
6
7
8
t

Results


0.02
0.015
0.01
0.005
0
E_w^{VR}
0
1
2
3
4
5
6
7
8
t
$\beta = 0.0$
$\beta = 1.0$
$\beta = 2.0$
$\beta = 4.0$

Closure

Work to do, Further work

1. Assess the VR model with viscous simulations, comparisons of:
    1. Translational velocity U
    2. VR energy
    3. VR circulation
    4. Swirl velocity w
2. Assess the effect of w on ω_φ

Closure

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Closure

Questions/Conclusions

1. Swirl velocity interferes in the Energy balance, thus affecting the translational velocity U of the VR, as described in Saffman.
2. Swirl velocity w affects the xy VR flow field (ω_{φ}) through second order interactions.
3. How does a naturally formed VR simulation compares to a synthetic VR simulation as a test case for the assessment of the confined swirl VR model?

Thank you for your attention