Transient heating of an evaporating droplet with presumed time evolution of its radius

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Objective

- To develop analytical solutions of the equation describing droplet heating in the presence of evaporation, taking into account the moving boundary effect.

- To analyse these solutions by comparing the predictions with existing models and available experimental data

Transient heating of an evaporating droplet

Governing equation:

$$\frac{\partial T}{\partial t} = \kappa \left(\frac{\partial^2 T}{\partial R^2} + \frac{2}{R} \frac{\partial T}{\partial R} \right)$$

$$\left(k_l \frac{\partial T}{\partial R} + hT \right) \bigg|_{R=R_d(t)} = hT_g + \rho_l L \dot{R}_d(t)$$

$$T(t=0) = T_{d0}(R)$$

Previous models

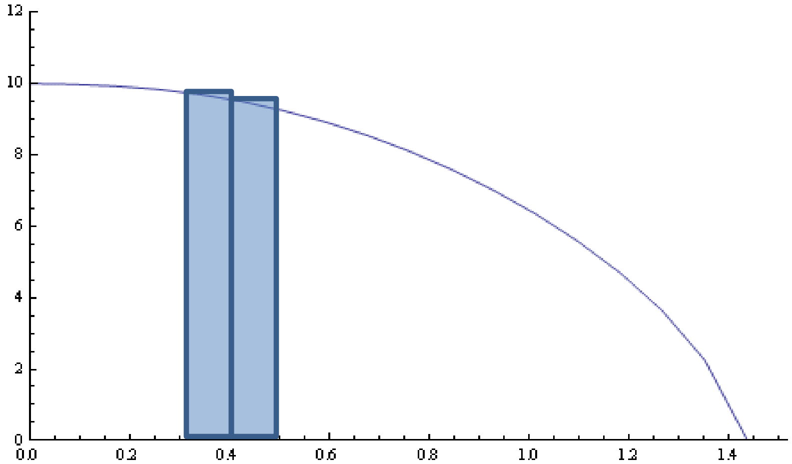
The conventional approach is based on the assumption that droplet radius remains constant during each timestep

$$Rd = Const$$

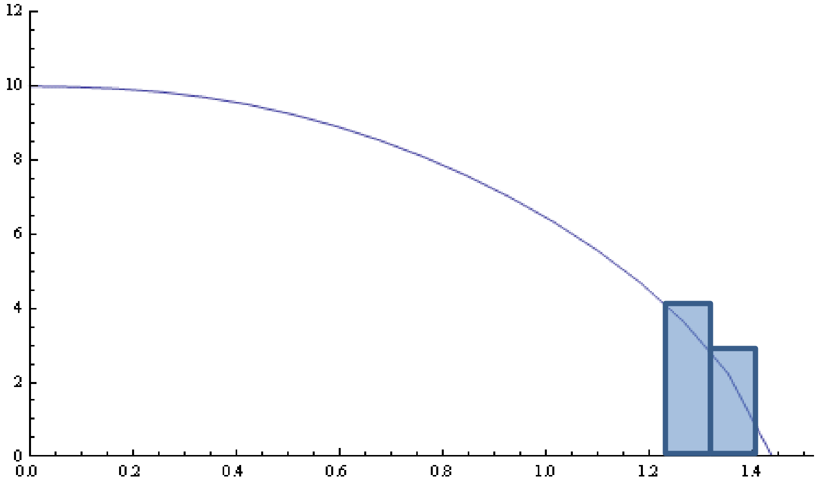
Problem: This assumption does not hold true for rapid evaporation

Model with Rd=Const

Everything is alright most of time



But at the end of the evaporation Rd changes too fast, to be approximated as a constant



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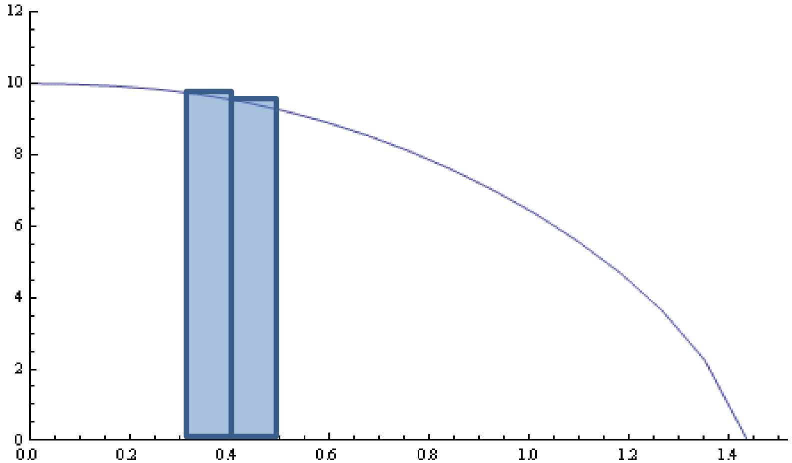

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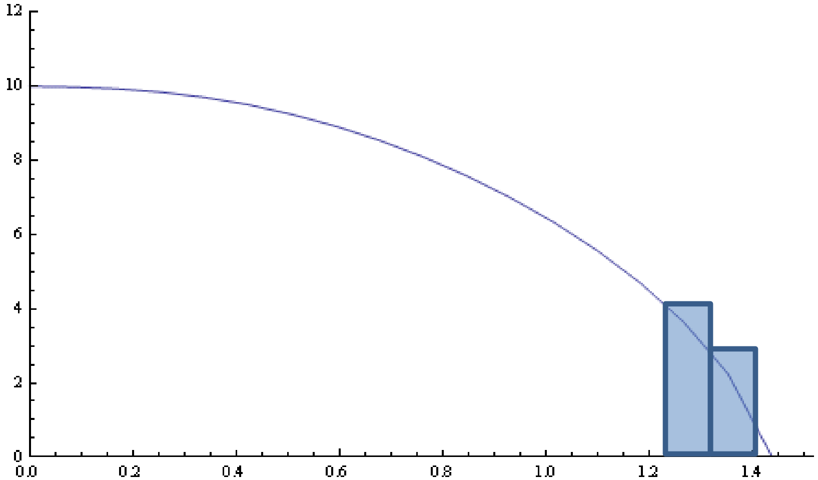


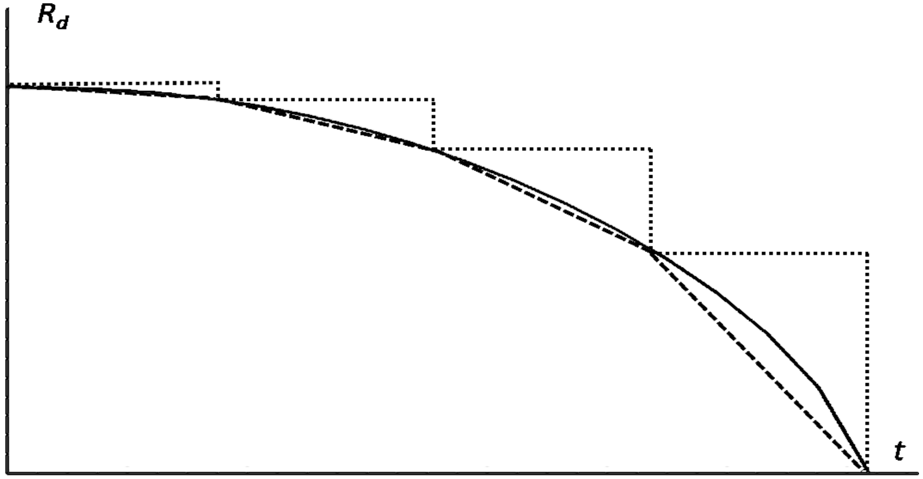
Obvious solution – "decreasing time step" will lead to increasing a computational time

Linear model

With

$$R_d(t) = R_{d0}(1 + \alpha t)$$

The problem at the end of the evaporation time disappeared



Analytical solution

$$T(R) = \frac{1}{R\sqrt{R_d(t)}} \exp\left[-\frac{\alpha R_{d0} R^2}{4\kappa R_d(t)}\right] \left[\sum_{n=1}^{\infty} \Theta_n(t) \sin\left(\lambda_n \frac{R}{R_d(t)}\right) + \frac{\mu_0(t)}{1+h_0} \frac{R}{R_d(t)}\right]$$

$$\Theta_n(t) = q_n \exp\left[-\frac{\kappa \lambda_n^2 t}{R_{d0} R_d(t)}\right] + f_n \mu_0(t)$$

$$-f_n \kappa \lambda_n^2 \int_0^t \frac{\mu_0(\tau)}{R_d^2(\tau)} \exp\left[\frac{\kappa \lambda_n^2}{\alpha R_{d0}} \left(\frac{1}{R_d(t)} - \frac{1}{R_d(\tau)}\right)\right] \mathrm{d}\tau.$$

$$f_n = \frac{1}{\| v_n \|^2} \int_0^1 f(\xi) v_n(\xi) \mathrm{d}\xi = -\frac{\sin \lambda_n}{\| v_n \|^2 \lambda_n^2}$$

$$\lambda \cos \lambda + h_0 \sin \lambda = 0$$

$$\| v_n \|^2 = \frac{1}{2}\left(1 - \frac{\sin 2\lambda_n}{2\lambda_n}\right) = \frac{1}{2}\left(1 + \frac{h_0}{h_0^2 + \lambda_n^2}\right)$$

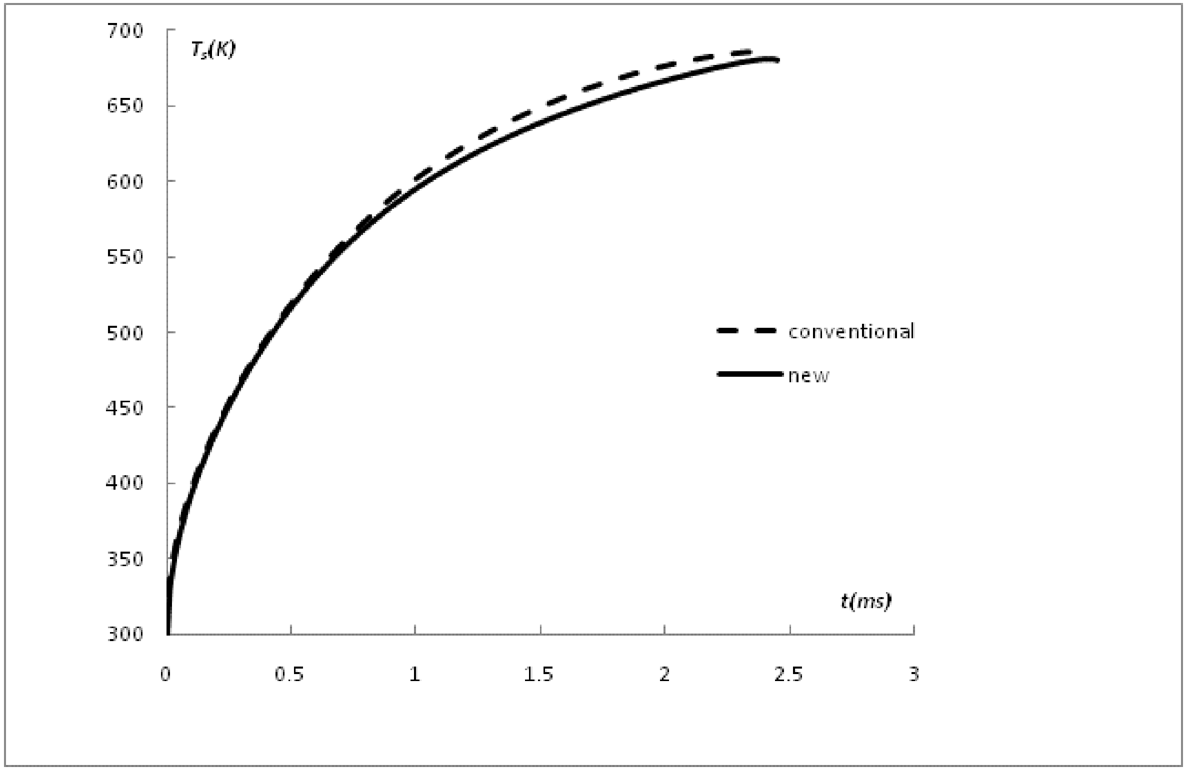
$$h_0 = \frac{h(t)}{k_l} R_d(t) - 1 - \frac{R_d'(t) R_d(t)}{2\kappa}$$

Linear model

Assuming that $R'_d=0$, the previous expression reduces to the one predicted by the model with R_d = const.

This means that the improved model includes the previous one.

Results



The plots of T_s versus time for heated and evaporating droplets using the conventional (dashed), and new (solid) approaches. The time step 0.008 ms was used for the conventional approach while the time step 0.01 ms was used for the new approach ($M_a = 29$ kg/kmole, $M_f = 170$ kg/kmole ($C_{12}H_{26}$), $p = 30$ atm $= 3000$ kPa, $T_g = 1000$ K, $R_{d0} = 10$ μm).

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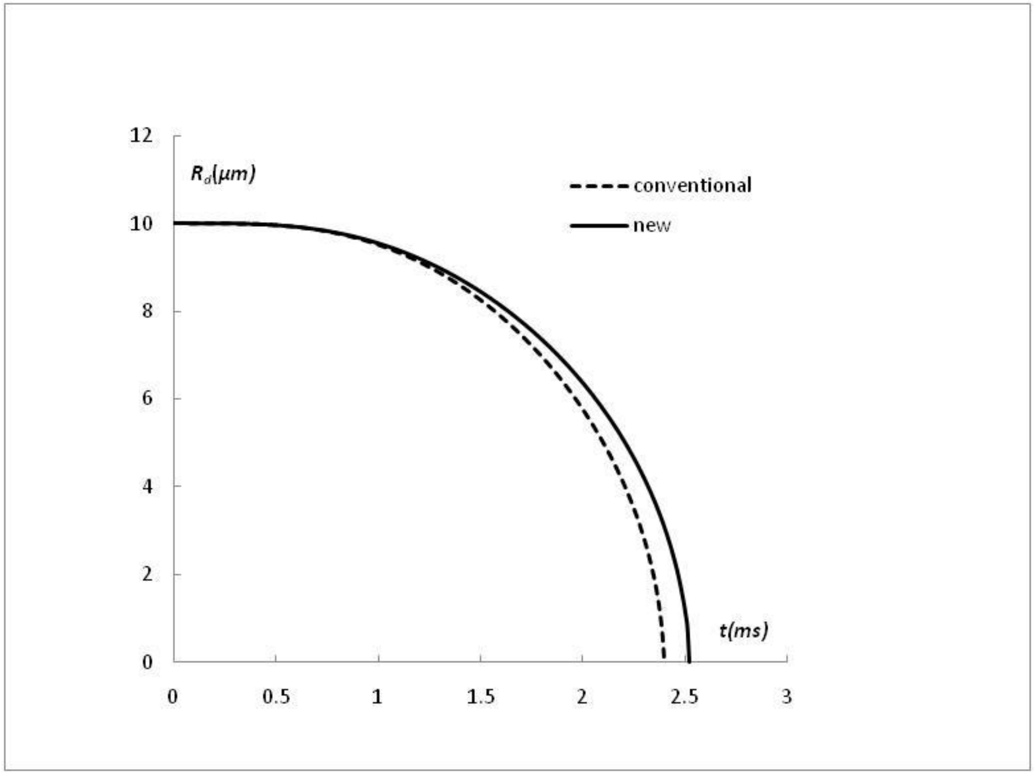

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Integral model

The main concept of the new model is the assumption that the droplet radius as a function of time is known in the whole range:

The Rd(t) function is known analytically .

Transient heating of an evaporating droplet

Let us rewrite boundary condition in the form:

$$\left(\frac{\partial T}{\partial R}+\frac{h}{k_l}T\right)\bigg|_{R=R_d(t)}=\frac{h}{k_l}T_g+\frac{\rho_l}{k_l}L\dot{R}_d(t)\equiv M(t)$$

and introduce the new variable $u = TR$

Transient heating of an evaporating droplet

The equation under new notations:

$$\frac{\partial u}{\partial t} = \kappa \frac{\partial^2 u}{\partial R^2},$$

$$\left(\frac{\partial u}{\partial R} + H(t) u \right) \Bigg|_{R=R_d(t)} = \mu(t),$$

$$u|_{R=0} = 0,$$

$$u(R)|_{t=0} = R T_{d0}(R),$$

where:

$$H(t) = \frac{h(t)}{k_l} - \frac{1}{R_d(t)}, \qquad \mu(t) = M(t) R_d(t).$$

Case $T_{d0}(R) = T_{d0} = \text{const}$

Let us introduce new variable: $v = u - RT_{d0}$

$$\frac{\partial v}{\partial t} = \kappa \frac{\partial^2 v}{\partial R^2},$$

$$\left(\frac{\partial v}{\partial R} + H(t) v \right)\Bigg|_{R=R_d(t)} = \mu_0(t),$$

$$v|_{R=0} = 0 \ , \quad v|_{t=0} = 0 \quad , \text{where}$$

$$\mu_0(t) = -T_{d0} - H(t) R_d(t) T_{d0} + \mu(t) = -\frac{h(t)}{k_l} R_d(t) T_{d0} + \mu(t).$$

Integral model

We look for the solution of the problem in the form:

$$v(R,t) = \int_0^t \nu(\tau) G(t,\tau,R)\,\mathrm{d}\tau,$$

where

$$G(t,\tau,R) = G_0(t-\tau, R - R_d(\tau)) - G_0(t-\tau, R + R_d(\tau)),$$

$$G_0(t,x) = \frac{\sqrt{\kappa}}{2\sqrt{\pi t}} \exp\left[-\frac{x^2}{4\kappa t}\right].$$

$$\left.\frac{\partial G(t,\tau,R)}{\partial R}\right|_{R=R_d(t)} \propto O\left(\frac{1}{\sqrt{t-\tau}}\right).$$

Integral model

Note that $G(t, \tau, R = 0) = 0$. $\nu(t)$ is an unknown continuous function to be found later from one of the boundary conditions. Function $v(R, t)$ is known as a single layer heat potential and it has the following properties for any continuous function $\nu(t)$:

1) It satisfies main equation;
2) It satisfies initial conditions;
3) It is continuous at $R \to R_d - 0$;
4) For the derivative $\partial v(R, t)/\partial R$ the following limiting formula is valid:

$$\left.\frac{\partial v(R,t)}{\partial R}\right|_{R \to R_d(t)-0} = \frac{\nu(t)}{2} + \int_0^t \nu(\tau) \left.\left[\frac{\partial G(t,\tau,R)}{\partial R}\right]\right|_{R=R_d(t)} \mathrm{d}\tau.$$

Integral model

$$\frac{\nu(t)}{2} + \int_0^t \nu(\tau) \left\{ \left[\frac{\partial G(t,\tau,R)}{\partial R} \right] \Bigg|_{R=R_d(t)} + H(t) G(t,\tau,R_d(t)) \right\} \mathrm{d}\tau = \mu_0(t),$$

$$T(t,R) = T_{d0} + \frac{\sqrt{\kappa}}{2R\sqrt{\pi}} \int_0^t \frac{\nu(\tau)}{\sqrt{t-\tau}} \left\{ \exp\left[-\frac{(R - R_d(\tau))^2}{4\kappa(t-\tau)} \right] - \exp\left[-\frac{(R + R_d(\tau))^2}{4\kappa(t-\tau)} \right] \right\} \mathrm{d}\tau.$$

Arbitrary $T_{d0}(R)$

$$T(t,R)=\frac{1}{R}\left[U(t,R)+\frac{\sqrt{\kappa}}{2\sqrt{\pi}}\int_0^t\frac{\nu(\tau)}{\sqrt{t-\tau}}\left\{\exp\left[-\frac{(R-R_d(\tau))^2}{4\kappa(t-\tau)}\right]-\exp\left[-\frac{(R+R_d(\tau))^2}{4\kappa(t-\tau)}\right]\right\}\mathrm{d}\tau\right]$$

$$\frac{\nu(t)}{2}+\int_0^t\nu(\tau)\left\{\left[\frac{\partial G(t,\tau,R)}{\partial R}\right]\Bigg|_{R=R_d(t)}+H(t)G(t,\tau,R_d(t))\right\}\mathrm{d}\tau=\mu_0(t),$$

$$G(t,\tau,R)=\frac{\sqrt{\kappa}}{2\sqrt{\pi(t-\tau)}}\left\{\exp\left[-\frac{(R-R_d(\tau))^2}{4\kappa(t-\tau)}\right]-\exp\left[-\frac{(R+R_d(\tau))^2}{4\kappa(t-\tau)}\right]\right\}$$

$$\mu_0(t)=\frac{1}{4\sqrt{\pi}(\kappa t)^{3/2}}\int_0^{R_{\mathrm{eff}}}(\zeta T_{d0}(\zeta))\left\{(R_d(t)-\zeta)\exp\left[-\frac{(R_d(t)-\zeta)^2}{4\kappa t}\right]\right.$$

$$\left.-(R_d(t)+\zeta)\exp\left[-\frac{(R_d(t)+\zeta)^2}{4\kappa t}\right]\right\}\mathrm{d}\zeta$$

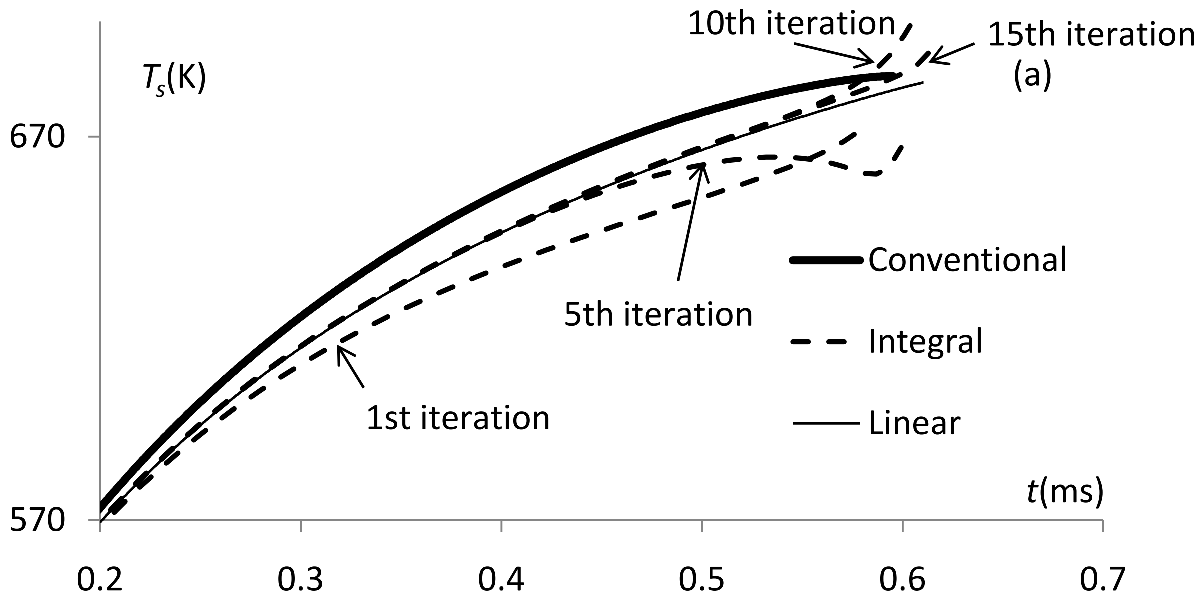
$$+\frac{H(t)}{2\sqrt{\pi\kappa t}}\int_0^{R_{\mathrm{eff}}}(\zeta T_{d0}(\zeta))\left\{\exp\left[-\frac{(R_d(t)-\zeta)^2}{4\kappa t}\right]-\exp\left[-\frac{(R_d(t)+\zeta)^2}{4\kappa t}\right]\right\}\mathrm{d}\zeta+M(t)R_d(t)$$

Integral model

Algorithm:

1) Calculate Rd(t) using conventional model
2) Approximate Rd(t) with polynomial approximation
3) Calculate T(R,t) using Rd(t) from 2nd step
4) Calculate Rd(t) using T(R,t) from 3rd step

Integral model - Results



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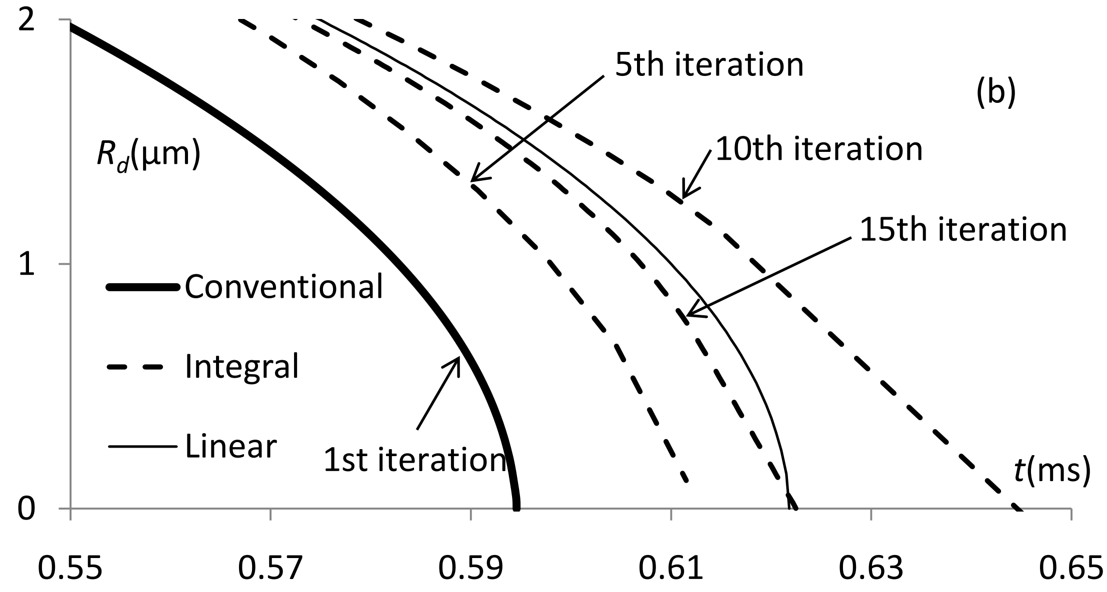

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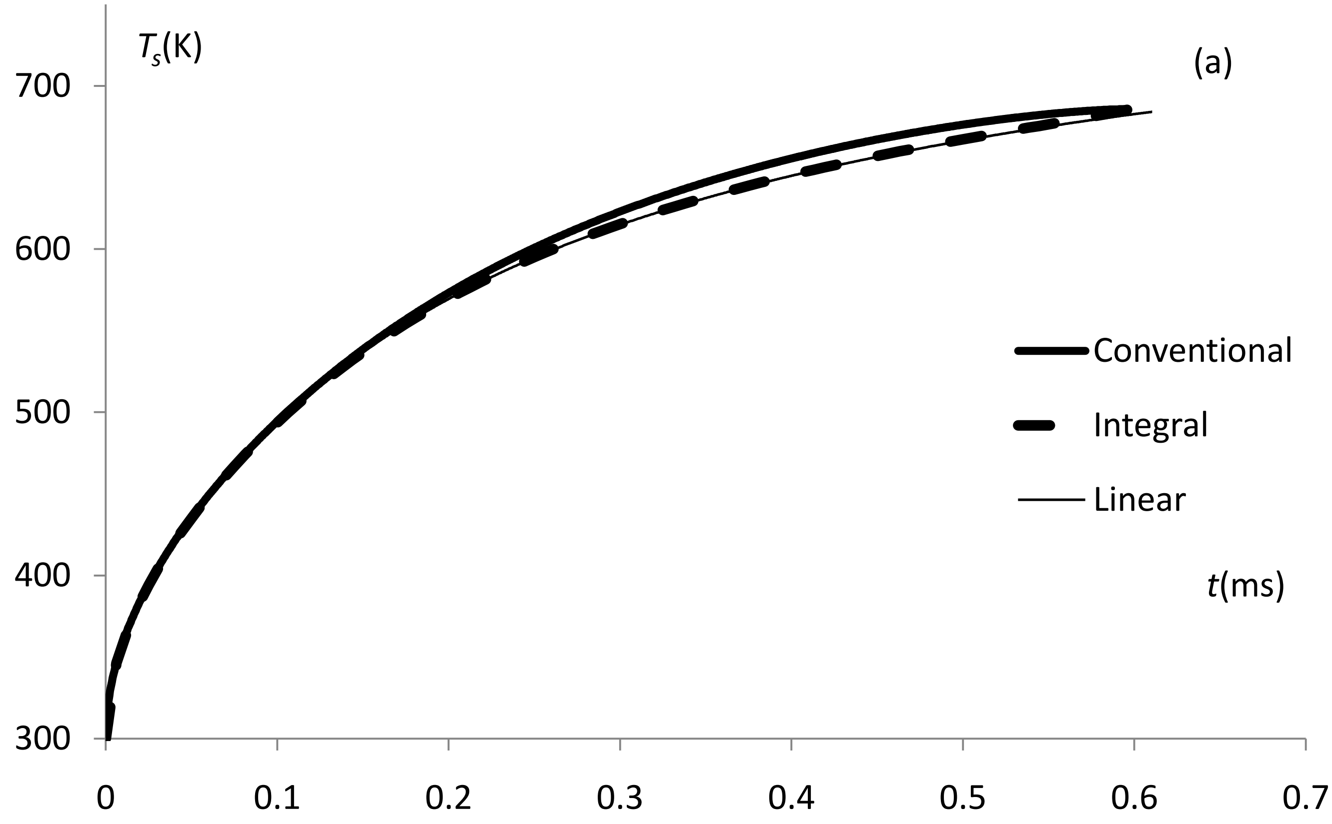
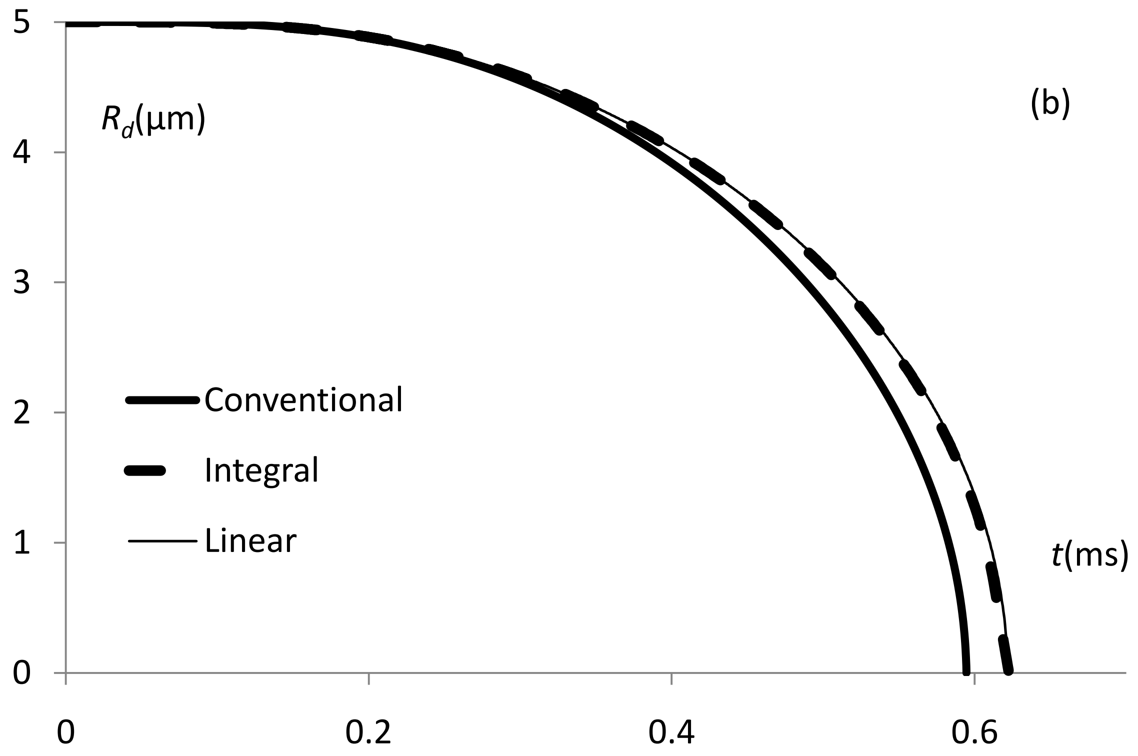


Integral model - Results



Conclusions (effects of moving boundary)

A new analytical solution to the heat conduction equation, describing transient heating of an evaporating droplet, is suggested. This solution takes into account the effect of the reduction of the droplet radius due to evaporation, assuming that this radius is a linear function of time. The solution has been incorporated into the zero dimensional CFD code and applied to the analysis of Diesel fuel droplet heating and evaporation in typical engine conditions. The new approach leads to the prediction of lower droplet temperatures and longer evaporation times than the traditional method.

Next steps

- Results with another models and experimental data
- Take into account droplet movement, thermal dilatation, radiation and coupling between liquid and gas phases
- Implement solution into a CFD code

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Thank you

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