Accelerating start-up pipe flows

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- Development of the Boundary Layer
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Start-up flow in pipes

- Szymanski (1932)
- Leutheusser, Lam (1977)
- Ohmi, Iguchi et al. (1980-1982)
- Kurokawa et al. (1985-1988)
- Das, Arakeri (1998)
- Zhao, Ghidaoui, Kolyshkin (2007)
- Nishihara, Knisley, Nakahata, Iguchi et al. (2007-2009)
Start-up flow
Development of the thickness of the boundary layer
Regions of laminar and turbulent flows

\[ \log U_{//}^* \]

\[ \log \alpha \]
Turbulence generation
Dissipative model

\[
\frac{\partial u}{\partial \tau} = -\frac{\partial q}{\partial \xi} + Dn \left( \frac{\partial^2 u}{\partial \eta^2} + \frac{1}{\eta} \frac{\partial u}{\partial \eta} \right),
\]

\[
\frac{\partial q}{\partial \eta} = 0
\]

\[
\frac{\partial q}{\partial \tau} + \frac{\partial u}{\partial \xi} + \frac{\partial v}{\partial \eta} + \frac{1}{\eta} v = 0,
\]
Flow with constant acceleration

\[ \frac{\partial U}{\partial \tau} = \alpha = \text{const.} \]

\[ \frac{\partial q}{\partial \xi} = -\alpha \left( 1 + \frac{4}{\sqrt{\pi}} \sqrt{\tau} + 3\tau + \frac{5}{\sqrt{\pi}} \tau \sqrt{\tau} \right). \]

\[ u(\mathcal{O}, \tau) = \alpha \int_{0}^{\tau} \left( 1 + \frac{4}{\sqrt{\pi}} \sqrt{\tau} \right) K(\mathcal{O}, \tau) d\tau \]

\[ K(\eta, \tau) = 1 - \frac{1}{\sqrt{\eta}} \left[ \left( 1 + \frac{(1-\eta)^2}{8\eta} \right) \text{erfc} \beta - \frac{1-\eta}{4\eta^{3/2}} \sqrt{\frac{\tau}{\pi}} e^{-\beta^2} + \ldots \right], \]
Velocity distribution
Radial velocity
Velocity distribution
Quasi-steady and ensemble-average wall shear stress
Thank You for Your Attention!