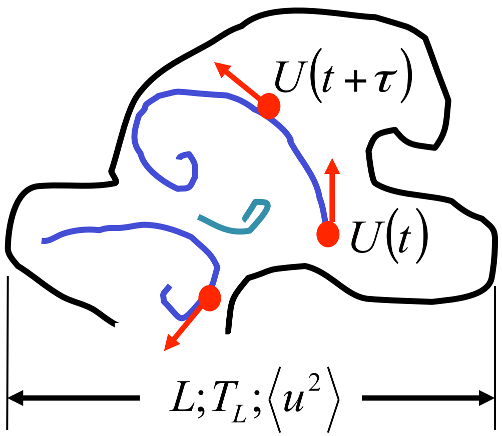
Lagrangian dispersion of light solid particle
in a high Re number turbulence;
LES with stochastic process at sub-grid scales

UMR 6614
CORIA
COMPLEXE DE RECHERCHE INTERPROFESSIONNEL EN AEROTHERMOCHIMIE
CNRS – UNIVERSITE et INSA de Rouen

Dispersion of fluid particle. Classical results



- **Taylor, 1921 :**

$$R^L(\tau) = \exp\left(-\frac{|\tau|}{T_L}\right) \qquad T_L = \int_0^\infty R^L(\tau)\,d\tau$$

$$D_2^L(\tau) = \left\langle \left(U_i(t+\tau) - U_i(t)\right)^2 \right\rangle = 2\langle u^2 \rangle \left(1 - R^L(\tau)\right)$$

- **Kolmogorov, 1941 :**

✓ dissipative zone ➡ $D_2^L(\tau) = \langle a^2 \rangle \tau^2 = a_0 \varepsilon^{3/2} \nu^{-1/2} \tau^2$

✓ inertial range ➡ $D_2^L(\tau) = C_0 \varepsilon \tau \approx \dfrac{2\langle u^2 \rangle \tau}{T_L}$

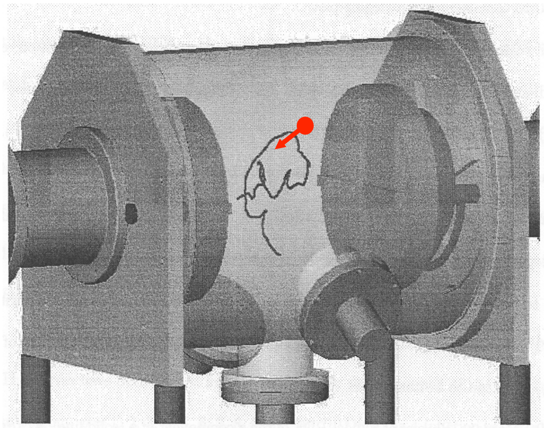
✓ large scales ➡ $D_2^L(\tau) = 2\langle u^2 \rangle$

✓ Gaussian distribution of particle velocity

Measurements of Lagrangian statistics of light particle in the high Re turbulence

(from Mordant and Pinton, ENS of Lyon, 2001, 2004)

Von Karman swirling flow of water



$\boxed{Re_\lambda = 740}$

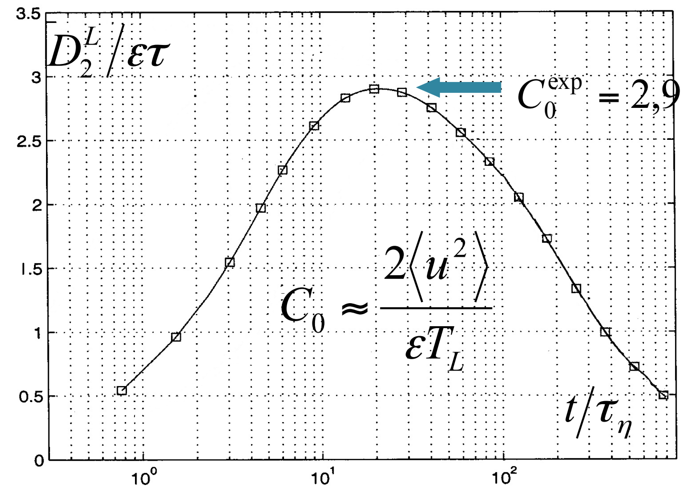
$\eta = 14\,\mu m$

$\tau_\eta = 0{,}2\,ms$

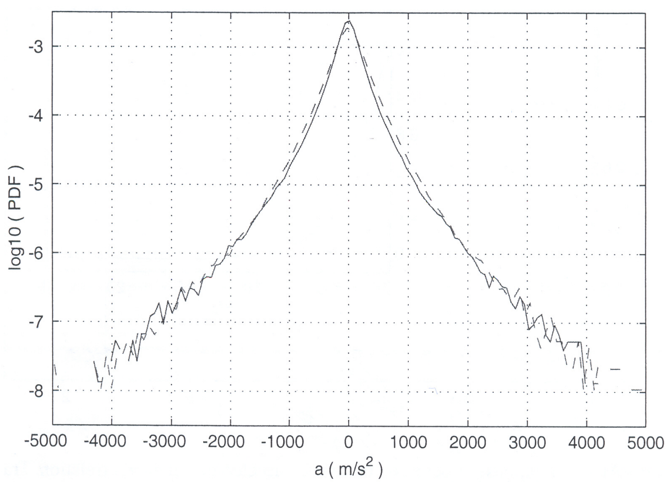
$\rho_p/\rho_f = 1{,}06$

$d_p = 250\,\mu m$

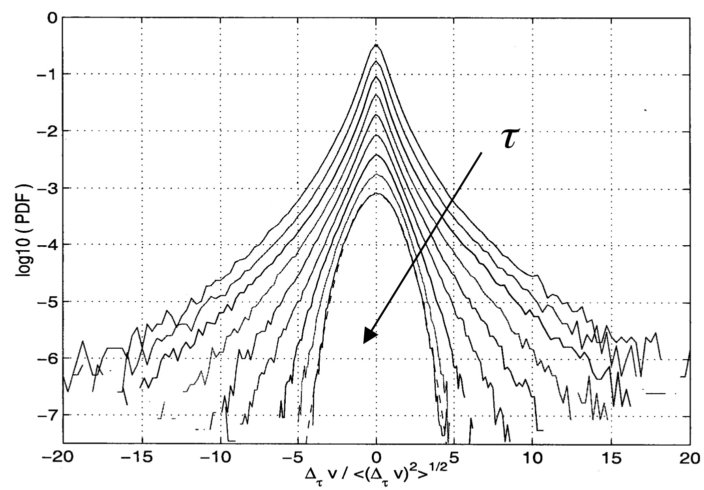
Second order structure function



PDF of particle acceleration

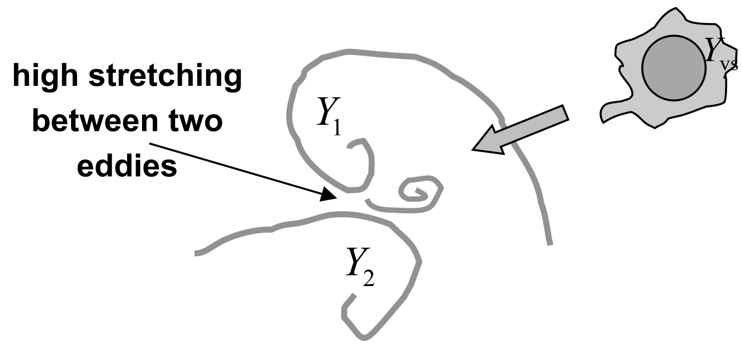


Normalized PDFs of the velocity increment



Importance of intermittency effects

In Diesels: Re is very high → flow is very intermittent at small scales



- **Strong local gradient of velocity:**

strong gradients of concentration,

“spontaneous” ignition or extinction sites,

burning spray extinction,

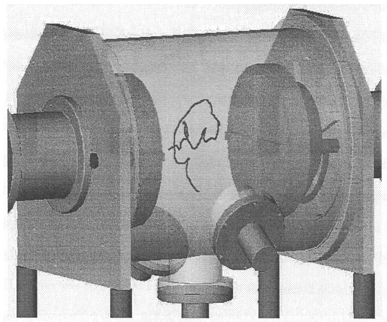
“preferential” concentration,

large spectrum of interacting spatial scales of liquid fragments

- **Need of droplet dynamics low in the highly intermittent flow !**

Our objective

$\mathrm{Re}_\lambda = 740$

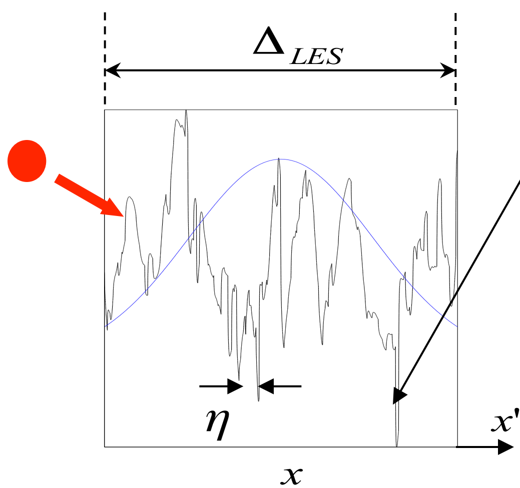


Compute this experiment

Problem:

the small-scale strong inhomogeneity is filtered

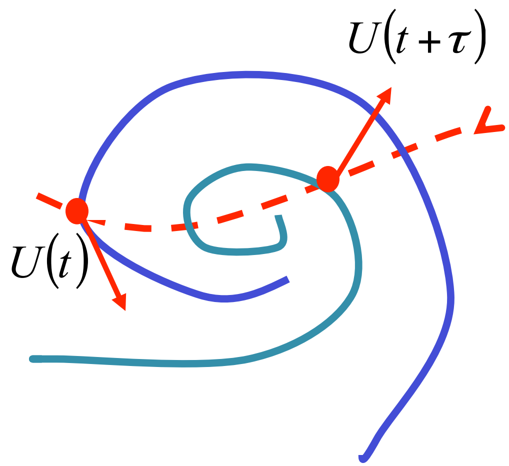
$$U(x') \xrightarrow{\text{LES}} U_g^{LES}(x) = \int_{-\infty}^{+\infty} U(x') G(x, x') dx'$$



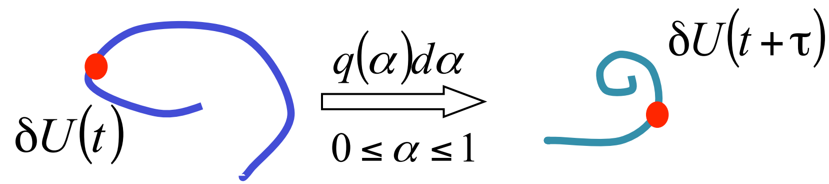
At sub-grid spatial scales, we need a model

$$U = U_g^{LES} + \delta U$$

Turbulent cascade as fragmentation under scaling symmetry



➢ Frequency of smaller eddies formation is high enough that
the portion of energy transmitted to the smaller eddy does
not depend on the energy of parent eddy
(scaling symmetry assumption)



$$|\delta U(t+\tau)| = \alpha |\delta U(t)| \Rightarrow \textit{evolution of } \; f(\delta U; t)$$

➢ At large times, this scenario reduces exactly to the Fokker-Planck-type equation

$$\frac{\partial f(\delta U;t)}{\partial t} = \frac{1}{T}\left[-\frac{\partial}{\partial(\delta U)}(\delta U)\langle \ln \alpha\rangle + \frac{1}{2}\frac{\partial}{\partial(\delta U)}(\delta U)\frac{\partial}{\partial(\delta U)}(\delta U)\langle \ln^2 \alpha\rangle\right] f(\delta U;t)$$

** Gorokhovski (2003) CTR, Stanford, Annual Briefs*
*** Gorokhovski & Chtab (2005), Lecture Notes in Computational Science and Engineering series, Springer*

Log-brownian stochastic process with constant force

- In the logarithmic space $q = \ln(\delta U)$ ---- usual form of Fokker-Planck equation

$$\frac{\partial Q(q;t)}{\partial t} = \frac{1}{T}\left[-\frac{\partial}{\partial q}\langle \ln \alpha \rangle + \frac{1}{2}\frac{\partial^2}{\partial q^2}\langle \ln^2 \alpha \rangle\right] Q(q;t)$$

- diffusivity / drift = ln(spatial scale) $\langle \ln^2 \alpha \rangle / \langle \ln \alpha \rangle = \ln\left(l_* / L_{ref}\right)$; $l_* = \eta$

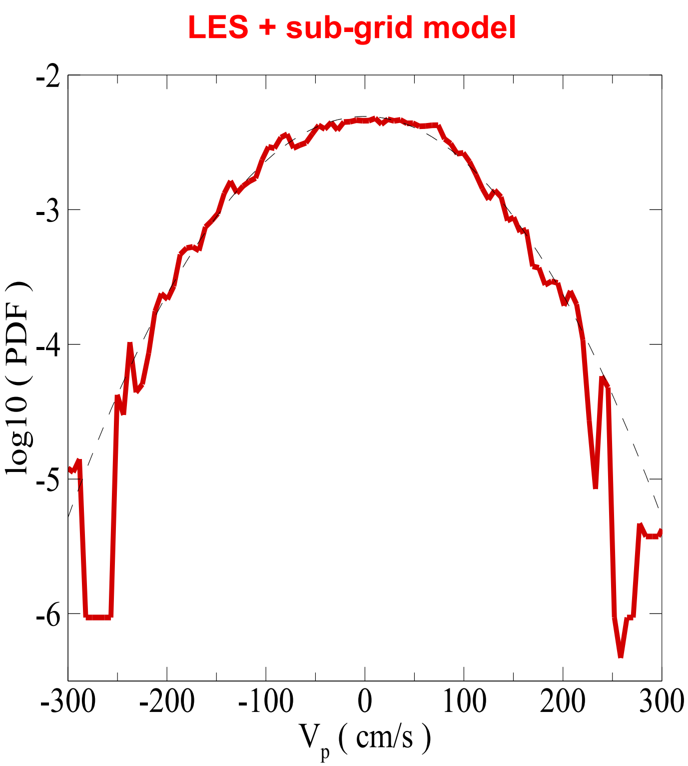
- Langevin stochastic equation : $$d \ln(\delta U) = \frac{\langle \ln \alpha \rangle}{T} dt + \sqrt{\frac{\langle \ln^2 \alpha \rangle}{2T}} dW(t)$$

- Finally : $$U_{local} = U^{LES} + \delta U$$

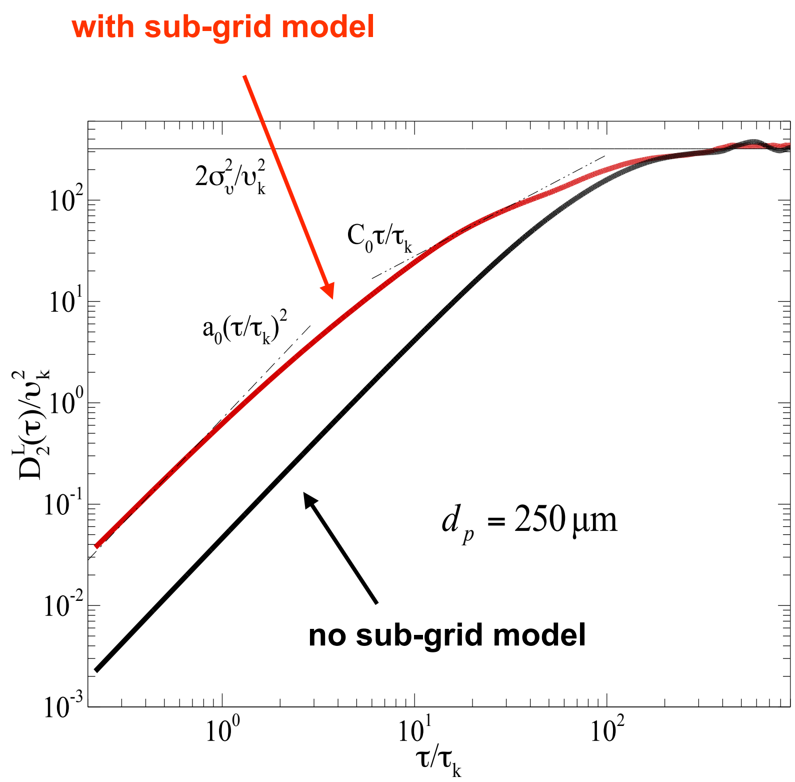
- Lagrangian tracking of solid particle : $$\frac{dV_p}{dt} = \frac{U_{local} - V_p}{\tau_{Stokes}}$$

Distribution of Lagrangian velocity : $V_p(t)$

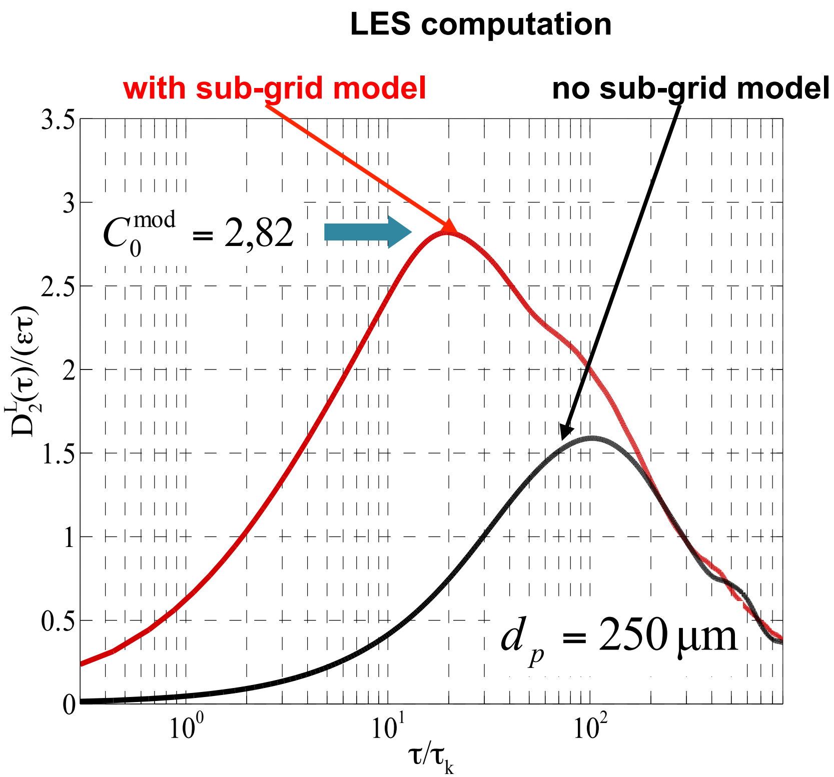
- velocity distribution follows Gaussian

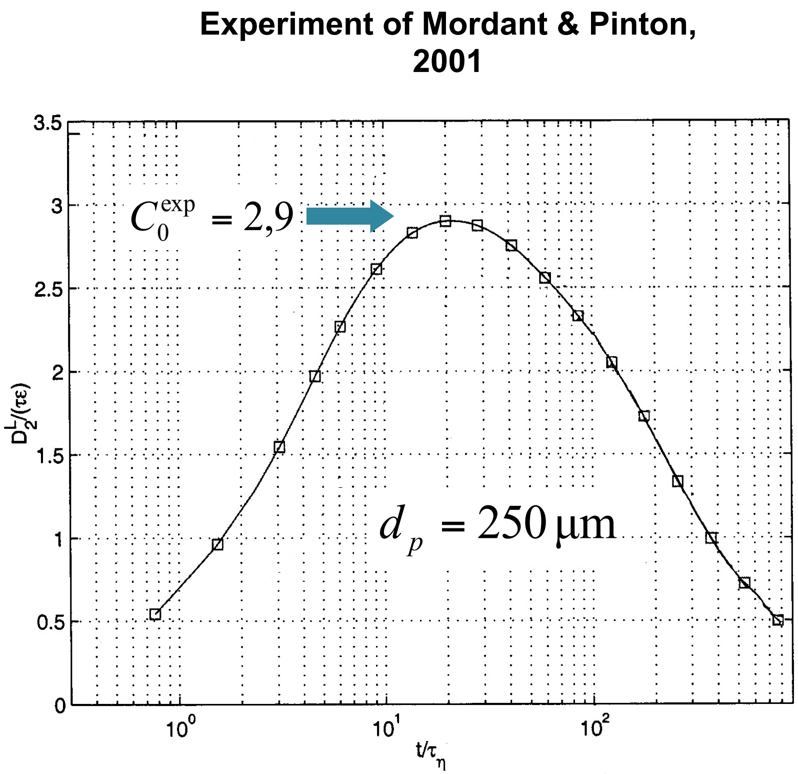


- inertial range is very restricted



Second order structure function : $D_2^L(\tau) = \left\langle \left(V_p(t+\tau) - V_p(t)\right)^2 \right\rangle$

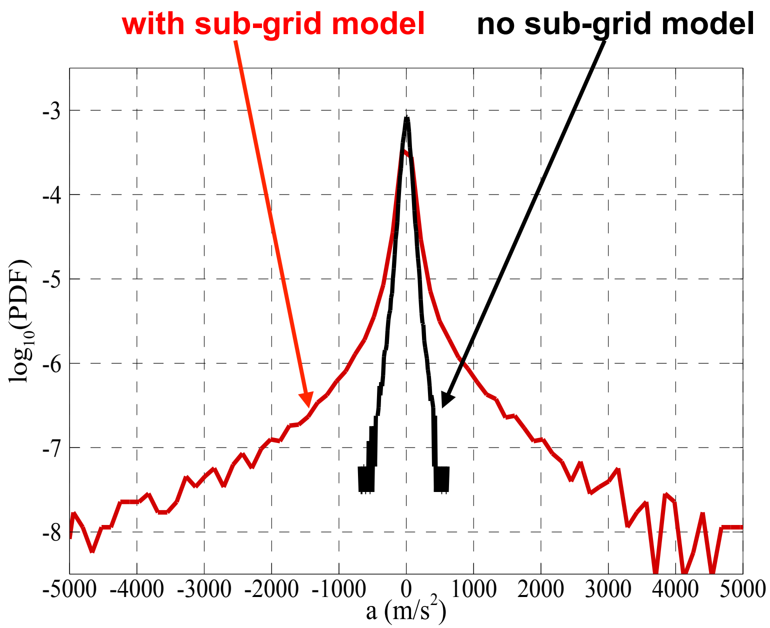




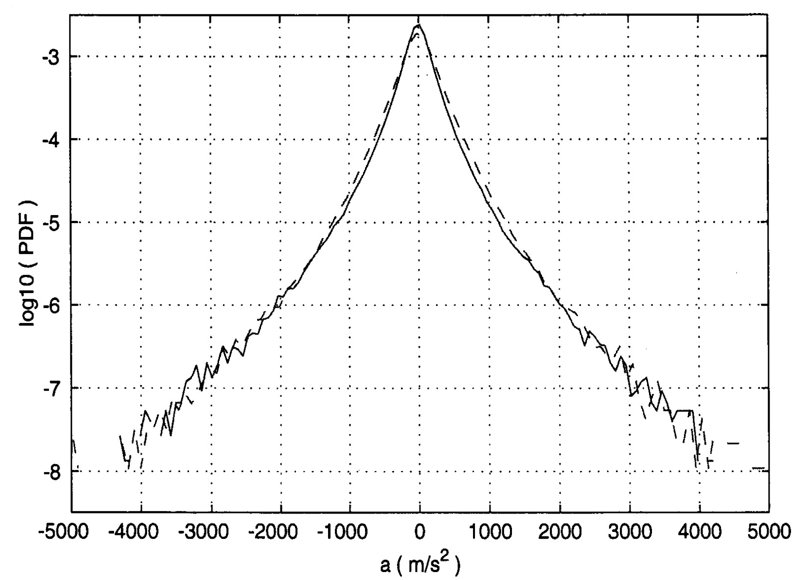
Distribution of Lagrangian acceleration

- from Kolmogov's scaling $a_k \propto \dfrac{\upsilon_k}{\tau_k} \approx 300\,\mathrm{m/s^2}$
- intermittency: weak accelerations alternated with "violent" accelerations

LES computation



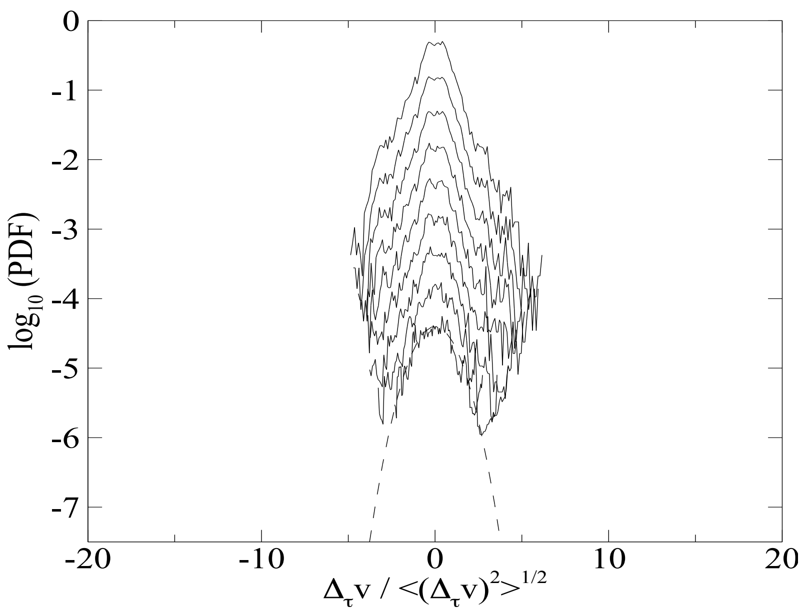
Experiment of Mordant & Pinton, 2001



Statistics of the velocity increment, $V_p(t+\tau)-V_p(t)$, at different time lags, τ

- at integral times ➡ Gaussian distribution
- at smaller times ➡ stretched tails with growing central peak

no sub-grid model



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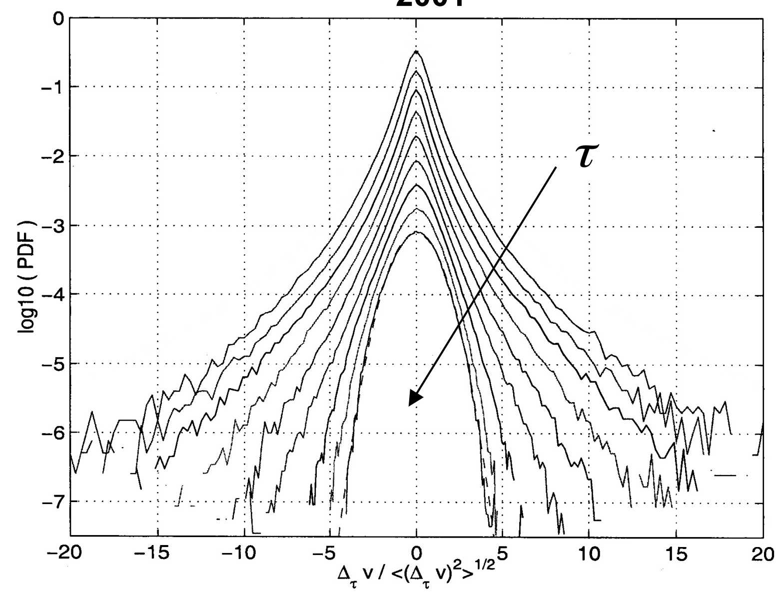

Conclusion

- Strong velocity gradients at small sub-grid scales

Stochastic modeling on sub-grid non-resolved scales

- LES computation + stochastic model + solid light particle tracking

$$U^{LES} + \delta U_{stoch} \longrightarrow \frac{dV_p}{dt} = \frac{U_{local} - V_p}{\tau_{Stokes}}$$

- Experimental observations of non-Gaussian behavior of solid light particle were reproduced in computation