Stochastic modeling of primary atomization : application to Diesel spray.

J. Jouanguy & A. Chtab & M. Gorokhovski
Introduction.

Atomization: main phenomena:
Turbulence in liquid and gas phase, cavitation, cycle by cycle variations

⇒ Deterministic description of such atomization is very difficult task.

⇒ Stochastic approach

⇒ Application to primary atomization.
Floating cutter particles.

Time $t_1$

Radial direction

Liq.

Axial direction

Time $t_2$

Radial direction

Liq.

Axial direction
The main assumption: scaling symmetry for thickness.

1. Scaling symmetry for thickness of liquid core.

\[ r(x,t+dt) = \alpha r(x,t) \]

\[ 0 < \alpha < 1 \]

Fragementation intensity spectrum

2. Life time.

3. Ensemble of \( n \) particles.

Evolution equation

\[ \frac{\partial f(r, t)}{\partial t} = v \int_0^1 f \left( \frac{r}{\alpha}, t \right) q(\alpha) \frac{d\alpha}{\alpha} - v f(r, t) \]

Knowledge of \( q(\alpha) \)

Fokker Planck type equation

\[ \frac{1}{v} \frac{\partial f(r, t)}{\partial t} = -\langle \ln \alpha \rangle \frac{\partial}{\partial r} (rf(t)) + \frac{\langle \ln^2 \alpha \rangle}{2} \frac{\partial^2}{\partial r^2} (rf(t)) \]

Knowledge of \( \langle \ln \alpha \rangle \) and \( \langle \ln^2 \alpha \rangle \)

Equation for the distribution function

Log brownian stochastic process

Langevin type equation

Equation for one realisation

\[ \xi = v \langle \ln \alpha \rangle r + \sqrt{v \langle \ln^2 \alpha \rangle / 2} r \Gamma(t) \]

\[ \frac{\langle \ln^2 \alpha \rangle}{\langle \ln \alpha \rangle} = \ln \left( \frac{r_c}{r_*} \right) \]

\( r_c \) = critical length scale

\[ \langle \ln \alpha \rangle = \text{CONST} \cdot \ln \left( \frac{r_c}{r_*} \right) \]

\( r_* \) = typical length scale
Identification of main parameters.

**Liquid**

\[ r_c = \text{critical radius} \quad r_{cr} \quad \rightarrow We_{cr} = \frac{\rho_g (u_g - u)^2 r_c}{\sigma} \]

\[ r_*= \text{Rayleigh Taylor scale} \quad \lambda_{RT} \]

**Time scale** \( \nu = \nu_{RT} \)

\[ \lambda_{RT} = 9.02 \left( \frac{(1 + 9.45Z^{0.5})(1 + 0.4T^{0.7})}{(1 + 0.87We_2^{1.67})^{0.6}} \right) r_0 \]

\[ We_1 = \rho \left( u_{g0} - u_{i0} \right)^2 r_0 / \sigma \]

\[ We_2 = \rho \left( u_{g0} - u_{i0} \right)^2 r_0 / \sigma \]

\[ \text{Re}_1 = \left( u_{g0} - u_{i0} \right) r_0 / \nu \]

\[ Z = We_1^{0.5} / \text{Re}_1 \]

\[ T = Z We_2^{0.5} \]

(Reitz 1987)
Realization of stochastic process; « Stochastic floating cutter particles ».

Motion of particles

In the radial direction:

\[ V_{ip} = \nu \langle \ln \alpha \rangle r + \sqrt{\nu \langle \ln^2 \alpha \rangle / 2} \ r \Gamma (t) \]

\[ \frac{dr}{dt} = V_{ip} \]

In the axial direction:

\[ \frac{dU_{ip}}{dt} = \frac{1}{2} \rho_s |u_g - u_{ip}| \frac{|u_g - u_{ip}|}{r} \]

\[ \frac{dx_{ip}}{dt} = U_{ip} \]

Statistics of liquid core boundary

\[ \Rightarrow \]
Experimental setup.


Gaz initial conditions:

Atmospheric conditions

\[ T = 300^\circ K \]
\[ p = 1 \text{bar} \]

Liquid initial conditions:

\[ \rho_p = 0.8g/cm^3 \]
\[ R_{inj} = 0.009cm \]
\[ t_{inj} = 0.85ms \]
\[ m_{inj} = 3.2mg \]
\[ T_{inj} = 300^\circ K \]
\[ U_{inj} = U_{inj}(t) = 0 \div 260m/s \]
Probability to get liquid core; formation of discreet blobs using presumed distribution:

Statistics of liquid core boundary

Injection of droplets

Radius:
\[ f(r) = \frac{1}{r_{typ}} \exp \left( -\frac{r}{r_{typ}} \right) \]

\[ r_{typ} \rightarrow \text{Radius of the injector} \]

Mass flow rate conservation

Motion of droplets

=> Standard KIVA procedure with velocity conditionned on the presence of liquid

\[ u_p = U_{inf}(t) \]

Initial conditions

\[ v_p = \frac{rad_p}{\tau} = \sqrt{K_{liquid} \frac{\rho_g}{\rho_p}} \]
Example of distribution of formed blobs.
Computed mean sauter diameter.

\[ D_{32} = \frac{\int D^3 f(D) dD}{\int D^2 f(D) dD} \]
Centerline droplet mean axial velocity.

Symbols = experiment

Line = simulation
Centerline Sauter Mean Diameter (SMD).

Symbols = experiment
Line = simulation
Application to Air-Blast atomization.

Experiment => mean liquid volume fraction (Stepowski & Werquin 2001)

Simulation => statistics of liquid core boundary

Key parameter: \[ M = \frac{\rho_g u_g^2}{\rho_l u_l^2} \]

Ug = 60 m/s, Ul = 1.36 m/s

Ug = 60 m/s, Ul = 0.68 m/s
Drop injection and lagrangian tracking.

Typical size resulting from primary atomization

\[ r_{typ} = \frac{1}{2} \left( \frac{\sigma}{\rho_g} We_{cr} \right)^{\frac{3}{2}} \epsilon^{-2/5} \]

Motion of the drops injected.

\[ \frac{dx_p}{dt} = u_p \]

Lagrangian tracking:

\[ \frac{du_p}{dt} = \frac{f}{St_p} \left( \langle u_g \rangle - u_p \right) \]

\( f \rightarrow \) Drag coefficient, \( St_p \rightarrow \) particle Stokes number

Modification of the gas velocity field:

\[ \langle u_g \rangle = u_g (1 - P_l) + u_{\text{ref}} P_l \]

\( P_l \rightarrow \) probability of presence of liquid
Distribution of formed blobs.

Experiments (Lasheras & al 1998)

\[ \text{U}_g = 140 \text{ m/s} \]

\[ \text{U}_l = 0.13 \text{ m/s} \]

\[ \text{U}_l = 2.8 \text{ m/s} \]

Examples of instantaneous distribution of formed droplets with instantaneous conditionned velocity of gaz and liquid core
Computation in the far field.

Secondary processes:

→ Shearing
→ Turbulence
→ Collisions

$S_{\text{eff}} = \pi (r_1 + r_2)^2$

Example of instantaneous distribution of formed droplets

$u_g = 140 \text{ m/s}, u_l = 0.55 \text{ m/s}$
Comparison in the far field.

\[ U_g = 140 \, \text{m/s} \]
Conclusion.

⇒ Simple engineering model for primary atomization is proposed.

⇒ This allows to form the blobs in the near-injector region.

Future work.

⇒ Comparison with experiments in Brighton (trying different main mechanisms for fragmentation).