

The problem of modelling of scale effects on cavitation flow is considered. A single-fluid model of cavitation, which takes into account the liquid quality and viscous shear stress effects, is proposed. The model is implemented into the computational fluid dynamics code PHOENICS and validated using available experimental data on cavitation flows in nozzles.

Similarity of cavitation flows

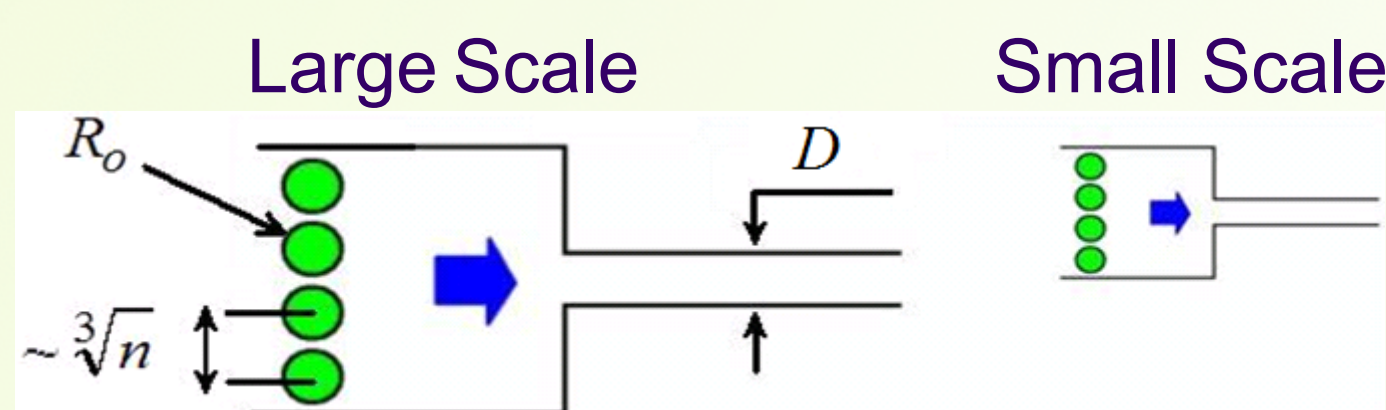
Similarity criteria

Reynolds number cavitation number Strouhal number

$$Re = \frac{\rho_l U \cdot D}{\mu_l} \quad CN = \frac{p_1 - p_2}{p_2 - p_v} \quad Str = \frac{f \cdot D}{U}$$

Scale effects

Cavitation bubble nuclei:



Radius and number density of cavitation bubbles give additional similarity criteria:

$$R_o/D; \quad \sqrt[3]{n}/D$$

Viscous scale effect:

Flow	Authors	Liquid	Images of the flow	p_1 (bar)	p_2 (bar)	$p_1 - p_2$ (bar)
Inlet cavitation	Roosen, et al. (1996)	Water		80	21	59
	Winklhofer, et al. (2001)	Diesel fuel		100	43	57
Super-cavitation	Roosen, et al. (1996)	Water		80	11	69
	Winklhofer, et al. (2001)	Diesel fuel		100	34	66

$p_1 - p_2 \sim 58 \text{ bar}$ (for inlet cavitation)
 $p_1 - p_2 \sim 68 \text{ bar}$ (for super-cavitation)

A model of hydrodynamic cavitation

Volume fraction equation

The liquid-vapour flow is described using the **homogeneous mixture** concept. The phase content and mixture properties are described by the vapour **volume fraction** α , governed by the transport equation:

$$\frac{\partial \alpha}{\partial t} + \frac{\partial u_j \alpha}{\partial x_j} = \frac{(1 - \alpha) \rho_l}{\rho} \cdot \frac{n}{1 + n \frac{4}{3} \pi R^3} \frac{d}{dt} \left(\frac{4}{3} \pi R^3 \right)$$

$$\alpha = \frac{n \frac{4}{3} \pi R^3}{1 + n \frac{4}{3} \pi R^3} \quad \text{volume fraction of vapour;}$$

$$\frac{dR}{dt} = \sqrt{\frac{2}{3}} \frac{|p_v - p_l|}{\rho_l} \cdot \text{sign}(p_v - p_l) \quad \text{-rate of the bubble growth/ collapse (Rayleigh, 1917);}$$

n - concentration of cavitation bubble nuclei, which has to be specified for particular cavitation flow, $1/m^3$.

The model was implemented into the CFD code PHOENICS. Discretisation of the void fraction equation was performed using the "super-bee" convection scheme (Hirsch, 1990).

Model for the concentration of bubble nuclei

A model for the parameter n has been derived to meet the similarity criterion:

$$C = \sqrt[3]{n}/D = \text{idem}$$

$$n = n^* \cdot \left(\frac{p_v - p_{\min}}{p_v} \right)^{3/2}$$

$p_v - p_{\min}$ = maximum tension in liquid, Pa;
 p_v = vapour pressure, Pa;
 n^* = adjustable liquid-specific number density parameter, $1/m^3$.

Model for the cavitation pressure threshold

Cavitation onset in a **static** liquid: $p < p_{cr} = p_v$

In a flowing liquid cavitation onset depends on **maximum tension** (Joseph, 1995):

$$-p + 2\mu S_{ii}^{\max} \geq -p_v$$

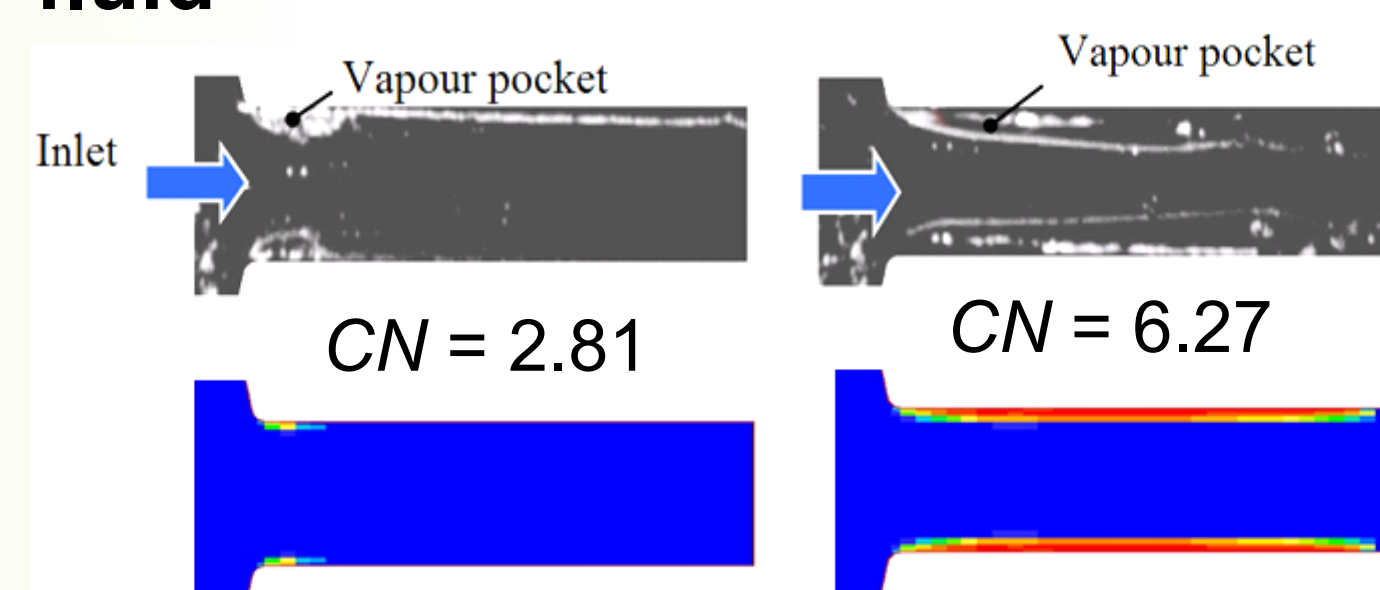
$$p < p_{cr} = p_v + 2\mu \cdot \left(1 + C_t \frac{\mu_t}{\mu} \right) \cdot S_{ii}^{\max}$$

S_{ii}^{\max} = maximal rate of strain, $1/s$;
 μ = dynamic viscosity of liquid, Pa s;
 μ_t = turbulent viscosity, Pa s;
 C_t = adjustable coefficient.

Results

Cavitation of a low viscosity fluid

Measured vapour-liquid field: (Roosen et al., 1996)

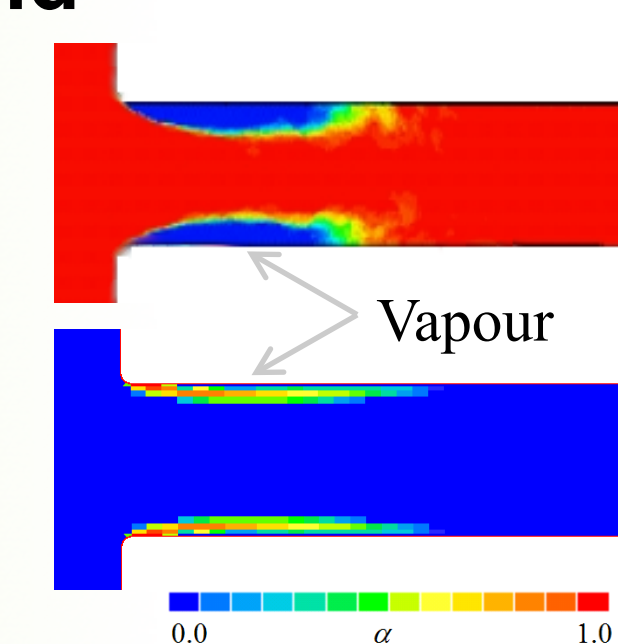


Numerical predictions:

$$n = 4.4 \cdot 10^{14} (m^{-3}); \quad C_t = 10$$

Cavitation of a high viscosity fluid

Measured vapour-liquid field: (Winklhofer et al., 2001)



Numerical predictions:

$$n = 2 \cdot 10^{18} (m^{-3}); \quad C_t = 10$$

Publications

- S. Martynov (2005) Numerical Simulation of the Cavitation Process in Diesel Fuel Injectors. Ph.D. thesis, University of Brighton, U.K.
- S. Martynov, D. Mason, and M. Heikal (2006) Numerical simulation of cavitation flows based on their hydrodynamic similarity. Int. J. Engine Research, 7 (3), pp. 283-296.

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Conclusions

- A homogeneous-mixture model of cavitation flow, based on the theory of bubble dynamics, has been extended in order to describe the liquid quality and viscous shear stress effects on cavitation flow.
- Assuming hydrodynamic similarity of cavitation flows, an algebraic model for the number density of active cavitation nuclei is suggested.
- The influence of viscous shear stress on cavitation flow has been clarified, and described in the model for the cavitation pressure threshold.
- The model was adjusted to describe sub-cavitation and super-cavitation flows in real-scale models of diesel injectors.