Dynamical effect of the coherent motion on small-scale statistics

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Turbulence phenomenon in Nature

✓ **Wide** and **continuous** range of scales

~~ Very large number of interactions between scales

✓ **Non-linearity**

~~ Sensitivity to the initial conditions

✓ Underlying **order** or **organization**

~~ Turbulence is not a purely random process

*Turbulence is one of the last mysteries of modern physics.*
✓ **K41** Asymptotic theory of turbulence \((Re \to \infty)\)
   - **Independence** between large and small scales
   - Locally **isotropic** context

✓ **Real** turbulence (finite \(Re\))
   - **Dependence** between large and small scales
   - Large scale **anisotropy** propagates down to the smallest scales
   - Populated by an **organized** motion

✓ **Organized motion**
   - Best prospect of being **controlled**.
   - **Persists** in space or time (long distance/time correlation)
   - Retains significant information about **initial conditions**
   - Strongly **anisotropic**

There is a gap between K41 and **real turbulence**, notably because the organized motion and its interactions with the small scales are not accounted for.

⇒ **Generalization** of Kolmogorov theory to account for **(1) finite Reynolds number effects**, the **(2) organized motion** and its **(3) impact on the small scales**.

\(^1\)Kolmogorov, A. 1941 Dokl. Akad. Nauk. SSSR 125 15-17
Exploring the interactions between CM and RM is important for:

✓ **Modeling** turbulent flows
    - → LES
    - → URANS
    - → DES models

✓ Building a strategy to **control** the activity of the **small scales**
    - → Micro-mixing
    - → Turbulent Combustion

✓ Controlling the large scales using perturbations at small scales.
    - → **Downsizing** actuators
    - → Decreasing their **consumption**
Questions

How to probe the activity of the small scales as a function of the activity of the CM?

What is the degree of the interactions between CM and the small scales:
- Production of turbulent kinetic energy
- Energy Transfer, Cascade
- Local isotropy, Universality

What are the scale-by-scale energy budget equations which account for the CM?

As a first step

Single type of coherent structures (wakes, shear layer, ...)

Quasi-periodic coherent motion
1 Introduction
   - Context
   - Fundamental and Practical Motivations
   - Questions

2 Effect of the CM on the small-scale activity
   - Phase-averaged structure function: Definition
   - Investigation in a cylinder wake flow
   - Results for second-order structure functions

3 Effect of the coherent motion on local isotropy
   - Phenomenological test of Isotropy

4 Scale-by-scale energy budget equations
   - Derivation
   - Investigation on the centerline of a wake flow

5 Conclusions
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5. Conclusions
Triple decomposition$^2$: $\beta = \beta + \tilde{\beta} + \beta'$

Phase-averaging operation: $\langle \beta \rangle = \beta + \tilde{\beta}$

Phase-averaged Strain: $\langle S' \rangle = \bar{S} + \tilde{S} = \frac{1}{2} \left( \frac{\partial \langle U \rangle}{\partial y} + \frac{\partial \langle V \rangle}{\partial x} \right)$

\[\Rightarrow\] Temporal dynamics associated with the coherent motion

---

$^2$Reynolds and Hussain 1972, J. Fluid Mech 54, 263-288
Total fluctuating field \( \langle (\Delta v)^2 \rangle = \langle [v(x + r) - v(x)]^2 \rangle \)

\[
\log_{10} \left[ \frac{\langle (\Delta v)^2 \rangle}{v^2} \right] (x = 10D)
\]

\[
\log_{10} \left[ \frac{\langle (\Delta v)^2 \rangle}{v^2} \right] (x = 20D)
\]

\[
\log_{10} \left[ \frac{\langle (\Delta v)^2 \rangle}{v^2} \right] (x = 40D)
\]
Coherent motion \( \langle \Delta \tilde{v} \rangle^2 = \langle v(x + r) - v(x) \rangle^2 \)

\[
\log_{10} \left[ \frac{(\Delta \tilde{v})^2}{\tilde{v}^2} \right] (x = 10D)
\]

\[
\log_{10} \left[ \frac{(\Delta \tilde{v})^2}{\tilde{v}^2} \right] (x = 20D)
\]

\[
\log_{10} \left[ \frac{(\Delta \tilde{v})^2}{\tilde{v}^2} \right] (x = 40D)
\]
Random motion $\langle (\Delta v')^2 \rangle = \langle (\Delta v)^2 \rangle - \langle \Delta \tilde{v} \rangle^2$

\[
\log_{10} \left[ \frac{\langle (\Delta v')^2 \rangle}{v'^2} \right] (x = 10D)
\]

\[
\log_{10} \left[ \frac{\langle (\Delta v')^2 \rangle}{v'^2} \right] (x = 20D)
\]

\[
\log_{10} \left[ \frac{\langle (\Delta v')^2 \rangle}{v'^2} \right] (x = 40D)
\]
**Conjecture:** "The effect of the turbulent strain rate at a scale $r$ must be much larger than the combined effect of the mean and coherent strain rates"\(^3\)

\[
s(r, \phi) \gg |\bar{S} + \tilde{S}|(\phi)
\]

- **Strain rate tensor:** $\Sigma = \nabla \vec{x} \vec{u}$
- **Strain rate acting on a scale $\vec{r}$ due to all larger scales\(^4\):**
  \[
  S\Sigma = \nabla \bar{\vec{x}} \bar{\vec{u}} + \nabla x^+ u^+
  \]
- $\bar{\vec{x}}$ and $x^+$ are independent:
  \[
  s(\vec{r}, \phi) = \left[\langle (S\Sigma)^2 \rangle \right]^{1/2} = \left[\langle (\nabla \vec{r} \Delta \bar{\vec{u}})^2 \rangle \right]^{1/2}
  \]
- After some manipulations:
  \[
  s(\vec{r}, \phi) = \left[ \frac{1}{2} \frac{\partial^2}{\partial r_j^2} \langle (\Delta u_i)^2 \rangle \right]^{1/2}
  \]

- In spherical coordinates:
  \[
  s(r, \phi) = \left[ \frac{1}{r} \frac{\partial}{\partial r} \langle (\Delta u_i)^2 \rangle + \frac{1}{2} \frac{\partial^2}{\partial r^2} \langle (\Delta u_i)^2 \rangle \right]^{1/2}
  \]


\(^4\)Mouri & Hori. Phys. Fluids 2010
**CENTERLINE**

\[
\log_{10}(\langle S \rangle/s_\phi) \quad (y = 0d)
\]

\[
\frac{\langle (\Delta v)^2 \rangle_{iso}}{\langle (\Delta v)^2 \rangle} \quad (y = 0d)
\]

**SHEARED REGION**

\[
\log_{10}(\langle S \rangle/s_\phi) \quad (y = 1.3d)
\]

\[
\frac{\langle (\Delta v)^2 \rangle_{iso}}{\langle (\Delta v)^2 \rangle} \quad (y = 1.3d)
\]
Resume

✓ Using Phase-averaged structure functions, it is shown that


\[ \text{Local anisotropy appears at rather large scales.} \]

\[ \text{Local anisotropy is dynamically related to the coherent strain rate} \]

✓ A **Phenomenological** test of isotropy is proposed


\[ \text{Based on the strain intensity at a given scale} \]

\[ \text{A } 10^{-1} \text{ criteria appears to be adapted to infer the anisotropic scale.} \]

✓ **Predictions** using scaling arguments


\[ \text{The ratio } s(r)/\langle \tilde{S} \rangle \propto Re_d^0. \]

\[ \text{The ratio } L_{\tilde{S}}/\lambda \propto Re_d^{1/2} \propto R_\lambda. \]

\[ \text{Persisting anisotropy in the far field } s(r)/|\langle S \rangle| \propto x^0 \text{ and } L_{\tilde{S}}/\lambda \propto x^0. \]
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**Navier-Stokes equation**

\[
\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = - \frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_j \partial x_j}
\]

**Triple decomposition + Phase-averaging**

\[
\beta = \bar{\beta} + \tilde{\beta} + \beta'
\]

\[
\langle \beta \rangle = \bar{\beta} + \tilde{\beta}
\]

**Dynamical equation** for the CM and the RM\(^5\)

\[
\frac{D\tilde{u}_i}{Dt} + \tilde{u}_j \frac{\partial \bar{U}_i}{\partial x_j} + \frac{\partial}{\partial x_j} (\tilde{u}_i \tilde{u}_j - \bar{u}_i \bar{u}_j) + \frac{\partial}{\partial x_j} \left( \langle u'_i u'_j \rangle - \bar{u}'_i \bar{u}'_j \right) = - \frac{\partial \tilde{p}}{\partial x_i} + \nu \frac{\partial^2 \tilde{u}_i}{\partial x_j \partial x_j},
\]

\[
\frac{Du'_i}{Dt} + \bar{u}_j \frac{\partial u'_i}{\partial x_j} + u'_j \frac{\partial \bar{U}_i}{\partial x_j} + u'_j \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial}{\partial x_j} (u'_i u'_j - \langle u'_i u'_j \rangle) = - \frac{\partial p'}{\partial x_i} + \nu \frac{\partial^2 u'_i}{\partial x_j \partial x_j}.
\]

\(^5\)Reynolds and Hussain 1972, J. Fluid Mech 54, 263-288
Written at points $\vec{x}$ and $\vec{x} + \vec{r}$ separated by a distance $\vec{r}$ and subtracted

**Dynamical equations** for the velocity increments (e.g. RM)

\[
\frac{\partial \Delta u'_i}{\partial t} + \Delta \left( \bar{U}_j \frac{\partial u'_i}{\partial x_j} \right) + \Delta \left( \tilde{u}_j \frac{\partial u'_i}{\partial x_j} \right) + \Delta \left( u'_j \frac{\partial \bar{U}_i}{\partial x_j} \right) + \Delta \left( u'_j \frac{\partial \tilde{u}_i}{\partial x_j} \right) + \Delta \left( \frac{\partial}{\partial x_j} \left( u'_i u'_j - \langle u'_i u'_j \rangle \right) \right) = -\Delta \left( \frac{\partial p'}{\partial x_i} \right) + \nu \Delta \left( \frac{\partial^2 u'_i}{\partial x_j^2} \right),
\]
Multiplication by e.g. $\Delta u'_i$, phase-averaging + time averaging

\[
\frac{D}{Dt} \Delta q'^2 + \frac{1}{X_j} \sum u'_j \Delta q'^2 + \sum \tilde{u}_j \langle \Delta q'^2 \rangle + 2 \Delta u'_i \Delta p' + 2 \Delta u'_i \Delta u'_j \frac{\partial U_i}{\partial x_j} + \frac{\partial}{\partial X_j} \sum u'_j \Delta u'_i \\
+ \frac{\partial}{\partial r_j} \left( \langle \Delta u_j \Delta q^2 \rangle - \Delta \tilde{u}_j \Delta \tilde{q}^2 \right) - 2 \Delta \tilde{u}_i \frac{\partial}{\partial r_j} \langle \Delta u'_i \Delta u'_j \rangle \\
- \nu \left[ \frac{\partial^2}{\partial r_j^2} + \frac{1}{2} \frac{\partial^2}{\partial x_j^2} \right] \Delta q'^2 + 2 \Sigma \left( \frac{\partial u'_i}{\partial x_j} \frac{\partial u'_j}{\partial x_i} \right) \right] = -2 \Sigma \tilde{c}^2.
\]
Effective energy transfer (divided by $\overline{\epsilon'r}$)

\[
\langle \Delta u \parallel \Delta q^2 \rangle - \Delta \tilde{u} \parallel \Delta \tilde{q}^2 - \frac{2}{r^2} \int_0^r \Delta \tilde{u}_i \frac{\partial}{\partial s} s^2 \langle \Delta u' \parallel \Delta u'_i \rangle ds
\]

![Graph showing effective energy transfer over scale $r/L_v$]
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