Modelling of the evolution of a droplet cloud in a turbulent flow field

Workshop: Vortex Rings and related processes

Brighton University
27th November 2015

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Modelling of the evolution of a droplet cloud in a turbulent flow field
Introduction

Aim of the current research is the "Development of the necessary tools and models for the investigation of the vortex ring-like structures arising during the fuel injection in internal combustion engines"

At a first stage the effects of confinement, swirl, heat transfer and the effect of the dispersed phase will be modelled utilising:

- Computational solution CFD of the carrier phase flow field. Using DNS for low Reynolds numbers and introducing an LES model for higher Reynold numbers.
- Full Lagrangian Approach (FLA) for the modelling of the particle transport.

In this presentation we investigate the applicability of the Full Lagrangian Approach (Osiptsov Method) in modelling particle dispersion for turbulent flows.

The presentation consists of the following parts.

- FLA and turbulence.
- DNS simulation of turbulent flows.
- Results on the evolution of particles conveying in Homogeneous and Isotropic turbulence.
- Closure.

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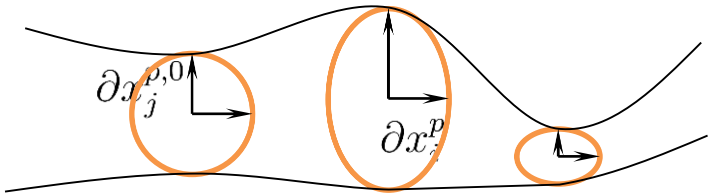
Full Lagrangian Aproach (FLA)



Concentration is calculated from the Lagrange form for parcel continuity

$$n_p(\mathbf{x_0};t) = \frac{n_p(\mathbf{x_0};0)}{|det(J_{ij})|} \ ,$$

utilizing the Jacobian of the transformation to the Eulerian coordinates

$$J_{ij} = \frac{\partial x_i^p}{\partial x_j^{p,0}} \ ,$$

The trajectory, the parcel velocities and the components of the Jacobian matrix are computed as a time advancement problem.

$$\frac{\partial x_i}{\partial \tau} = V_i \ ,$$

$$\frac{\partial V_i}{\partial \tau} = f_i \ . \qquad f_i = \frac{1}{St}(U_i - V_i)$$

$$\frac{\partial J_{ij}}{\partial \tau} = \omega_{ij} \ .$$

$$\frac{\partial \omega_{ij}}{\partial \tau} = \frac{1}{St}\left(J_{kj}\frac{\partial U_i}{\partial x_k} - \omega_{ij} \right) \ .$$

The auxiliary variable ω_{ij} is expressed using the definition of the Jacobian components. Assuming a Stokes expression for the drag acting on the particles the rate of ω_{ij} is expressed solely on Eulerian field variables.

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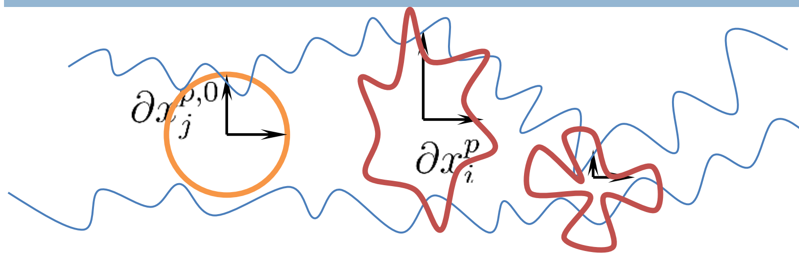
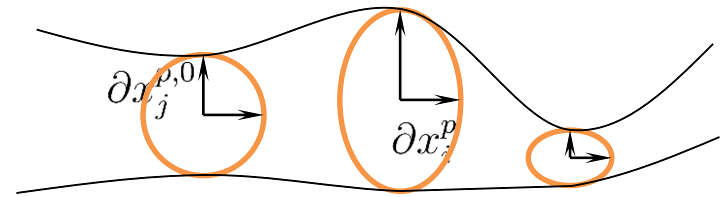
Full Lagrangian Aproach (FLA) in turbulence





The wide range of the length scale that describe the flow structures within a turbulent flow field prevent us from a direct solution of the NS for large Reynolds numbers. In such cases the problem is reduced into trying to solve the equations for the temporally or spatially averaged field variables.

In RANS the temporally averaged field variable $\overline{U}$ is solved instead

$$\overline{g}(\mathbf{x};t) = \frac{1}{T}\int_{t-T/2}^{t+T/2} g(\mathbf{x};t)dt \ ,$$

$$g(\mathbf{x};t) = \overline{g}(\mathbf{x};t) + g'(\mathbf{x};t) \text{ with } \overline{g'}(\mathbf{x};t) = 0$$

$$\begin{aligned}
\frac{\partial x_i}{\partial \tau} &= \overline{V}_i \\
\frac{\partial \overline{V}_i}{\partial \tau} &= \frac{1}{St}(\overline{U}_i - \overline{V}_i) \\
\frac{\partial \overline{J}_{ij}}{\partial \tau} &= \overline{\omega}_{ij} \\
\frac{\partial \overline{\omega}_{ij}}{\partial \tau} &= \frac{1}{St}\left(\overline{J}_{kj}\frac{\partial \overline{U}_i}{\partial x_k} - \overline{\omega}_{ij}\right)
\end{aligned}$$

$$\overline{J}_{ij} = \frac{\partial x_i^p}{\partial x_j^{p,0}} \ .$$

$$\overline{n}_p(\mathbf{x_0};t) = \frac{n_p(\mathbf{x_0};0)}{|det(\overline{J}_{ij})|} \ ,$$

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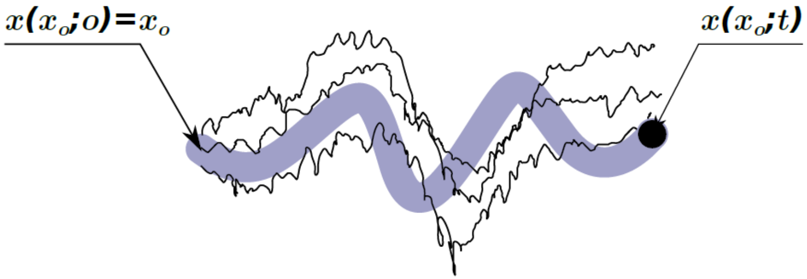
Full Lagrangian Aproach (FLA) in turbulence

The averaged path line is not expected to be contained and mass is exchanged through the unresolved turbulent scales

In cases of spatial gradients in concentration then the continuity equation provided by the FL approach should be enriched with a term accounting for turbulent diffusion



$$\overline{n}_p(\mathbf{x};t) = \frac{n_p(\mathbf{x_0};0)}{|det(\overline{J}_{ij})|}$$

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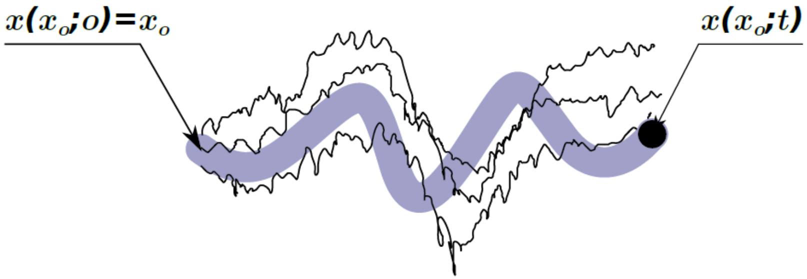

$$\overline{n}_p(\mathbf{x};t) = \frac{n_p(\mathbf{x_0};0)}{|det(\overline{J}_{ij})|} + \frac{1}{V_p^t}\left(\int_{S_p} F^{\mathrm{turb}}\vec{n}dS\right)dt \ ,$$

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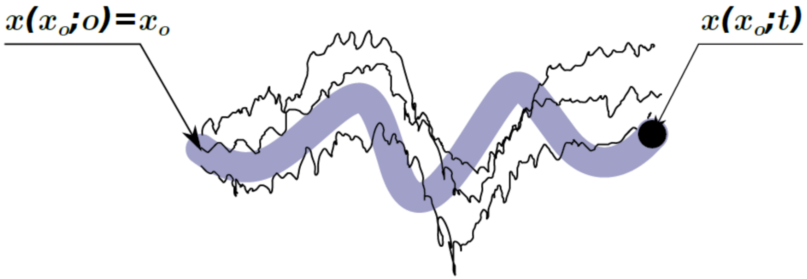

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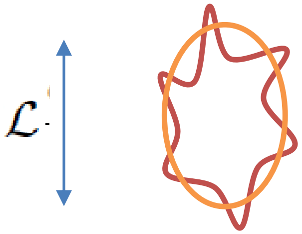

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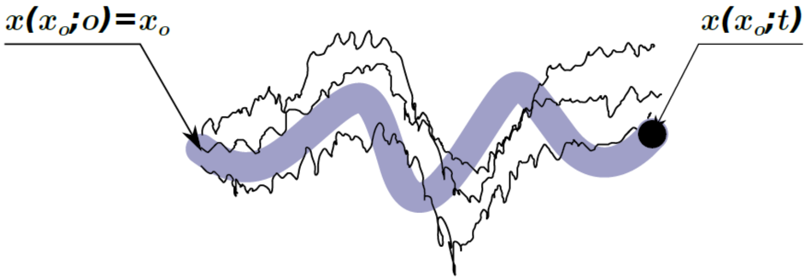

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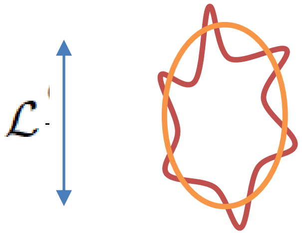

$$\int_S F\vec{n}dS = \frac{\tau_t}{\tau_p}\mathcal{U}\mathcal{L}\int_S \nabla n_p\vec{n}dS = \frac{\tau_t}{\tau_p}\mathcal{U}\mathcal{L}\int_V \Delta n_p dV = \frac{\tau_t}{\tau_p}\mathcal{U}\mathcal{L}\Delta n_p V_p^t\;,$$

$$\mathcal{V} \approx \frac{\tau_t}{\tau_p}\mathcal{U}\;,$$

For high inertia particles we could assume that particle fluctuations are a fraction of the turbulent fluctuations

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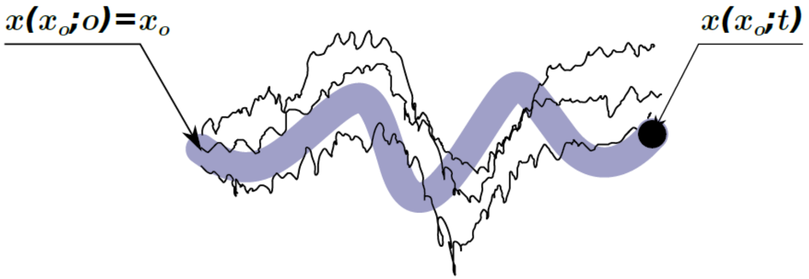
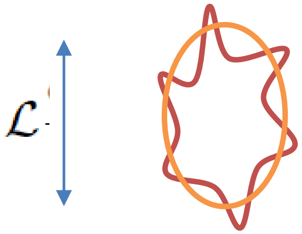


$$\overline{n}_p(\mathbf{x};t) = \frac{n_p(\mathbf{x_0};0)}{|det(\overline{J}_{ij})|} + \frac{1}{V_p^t}\left(\int_{S_p} F^{\mathrm{turb}}\vec{n}dS\right)dt \qquad F^{\mathrm{turb}} = \mathcal{V}\mathcal{L}\frac{dn_p}{dx}\ .$$



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$$\mathcal{V} = \mathcal{U}e^{-t_p/4t_t}$$

For stationary turbulence the particle fluctuations converge to a percentage of turbulent fluctuation that leads to 1 for low inertia particles

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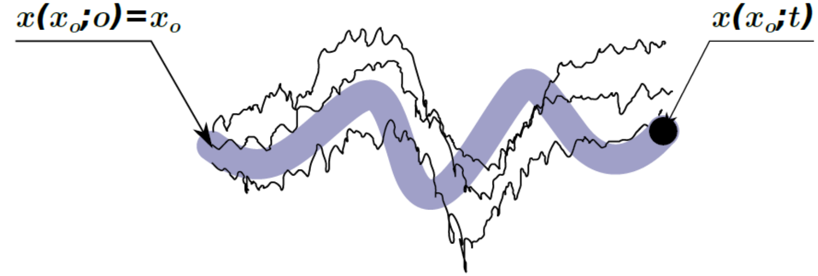
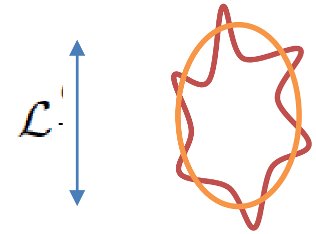


$$\overline{n}_p(\mathbf{x};t) = \frac{n_p(\mathbf{x_0};0)}{|det(\overline{J}_{ij})|} + \frac{1}{V_p^t}\left(\int_{S_p} F^{\mathrm{turb}}\vec{n}dS\right)dt \qquad F^{\mathrm{turb}} = \mathcal{VL}\frac{dn_p}{dx} \; .$$



$$\int_S F\vec{n}dS = \frac{\tau_t}{\tau_p}\mathcal{UL}\int_S \nabla n_p \vec{n}dS = \frac{\tau_t}{\tau_p}\mathcal{UL}\int_V \Delta n_p dV = \frac{\tau_t}{\tau_p}\mathcal{UL}\Delta n_p V_p^t \; ,$$

$$\dot{V}^2 = \frac{2}{t_p}\left[U^2 e^{-t_p/2t_t} - V^2\right]$$

Finally 1eq models should be used for modelling of the dynamic response of the particles. Crowe 77, Gosman 83

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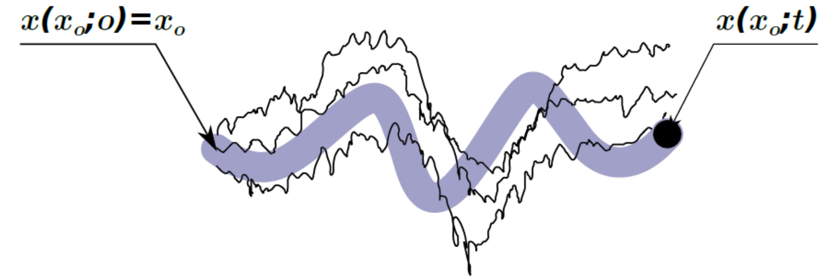

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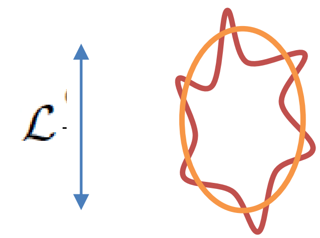

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$$\mathcal{V} \approx \frac{\tau_t}{\tau_p}\mathcal{U} \ , \qquad \overline{n_p} = \frac{n_p^0}{|\overline{J}|} + \frac{\tau_t}{\tau_p}\nu_t\Delta\overline{n}_p dt \ .$$

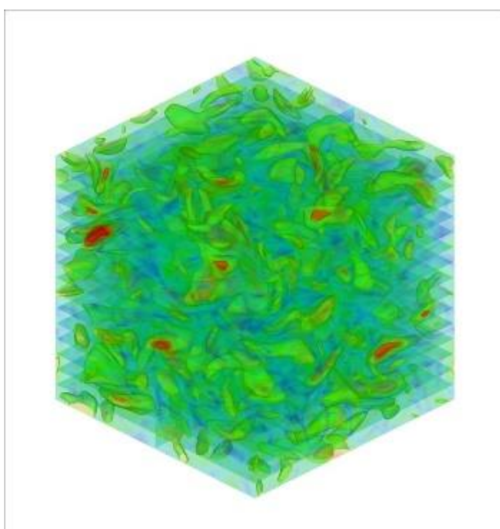
Modelling of the evolution of a droplet cloud in a turbulent flow field
Model Assumptions

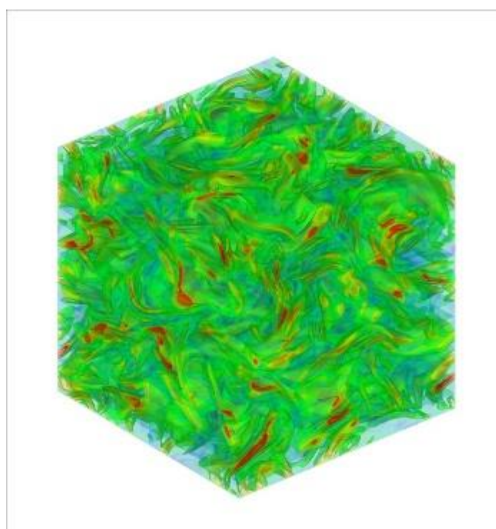
Scope of our research would be the assessment of the hypotheses introduced by the suggested model.

$$\overline{n_p} = \frac{n_p^0}{|\overline{J}|} + \frac{\tau_t}{\tau_p} \nu_t \Delta \overline{n}_p dt \; .$$

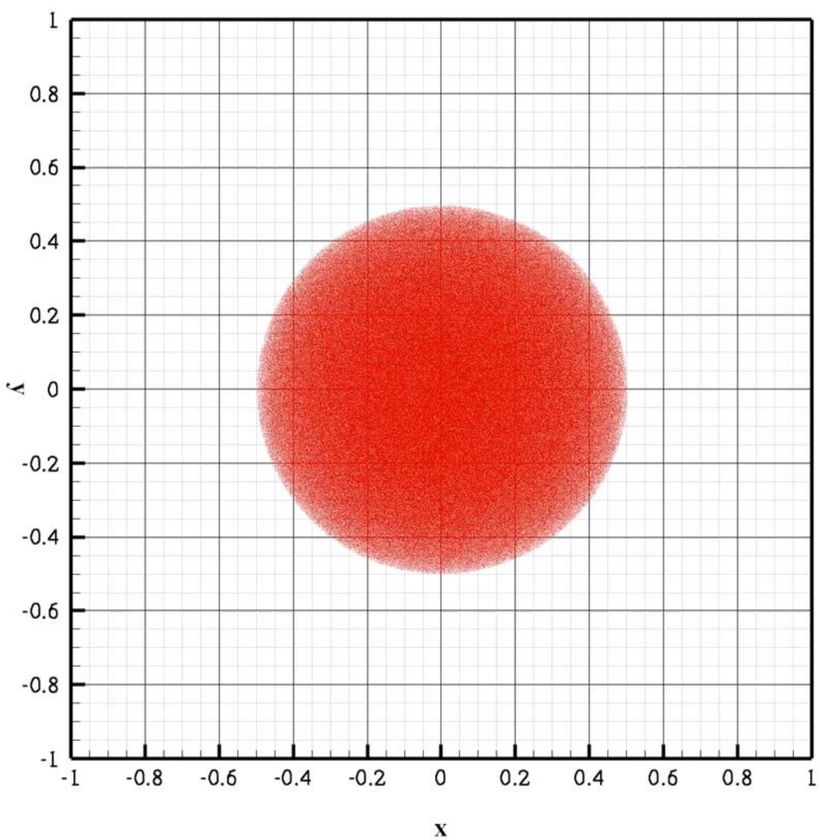
Synthetic turbulence vorticity

Forced DNS resulting to Homogeneous Isotropic field





At this stage direct numerical simulations of particle transport in homogeneous and isotropic turbulence have been carried out.



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Direct Numerical Simulation (DNS)

Direct simulations of turbulence are based on providing adequate mesh resolution that would accommodate a substantial range of turbulent scales.

By this way, the anisotropies of the integral scale, the turbulent cascade of the inertia scale and finally the viscus dumping of the small Kolmogorov scales are modelled.

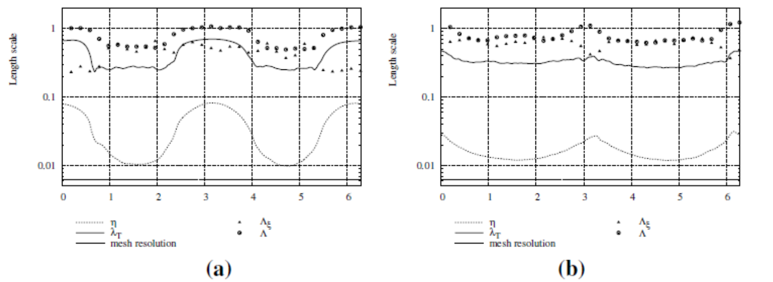
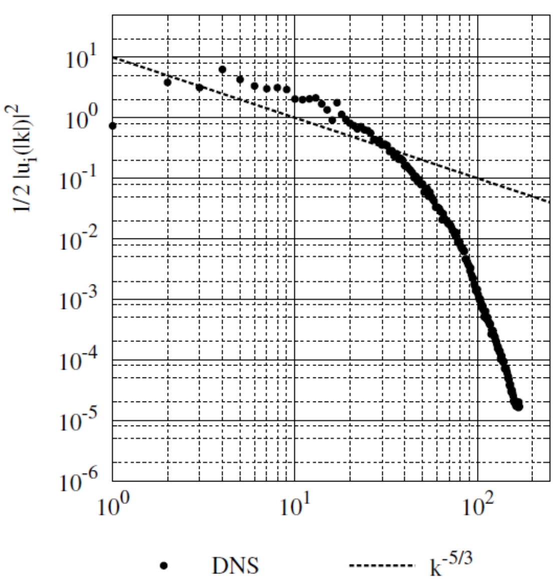
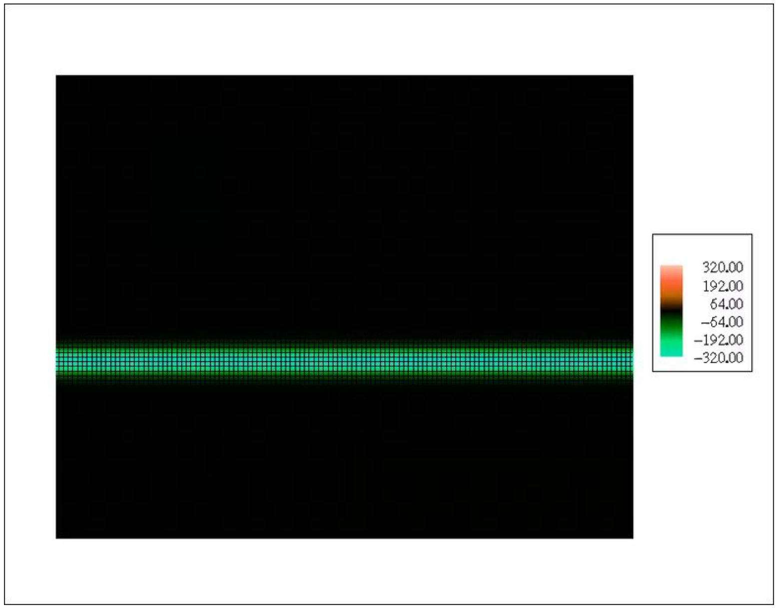


Figure 4.16: The distribution of the Integral length scale Λ, the integral length scale Λ_ξ for the mixture fraction distribution, the Taylor length scale λ_T, the Kolmogorov length scale η and the mesh resolution of the simulation for the case JMixing-B, across the planar jet. **(a)** at $t^* = 26.9$ **(b)** at $t^* = 60.0$





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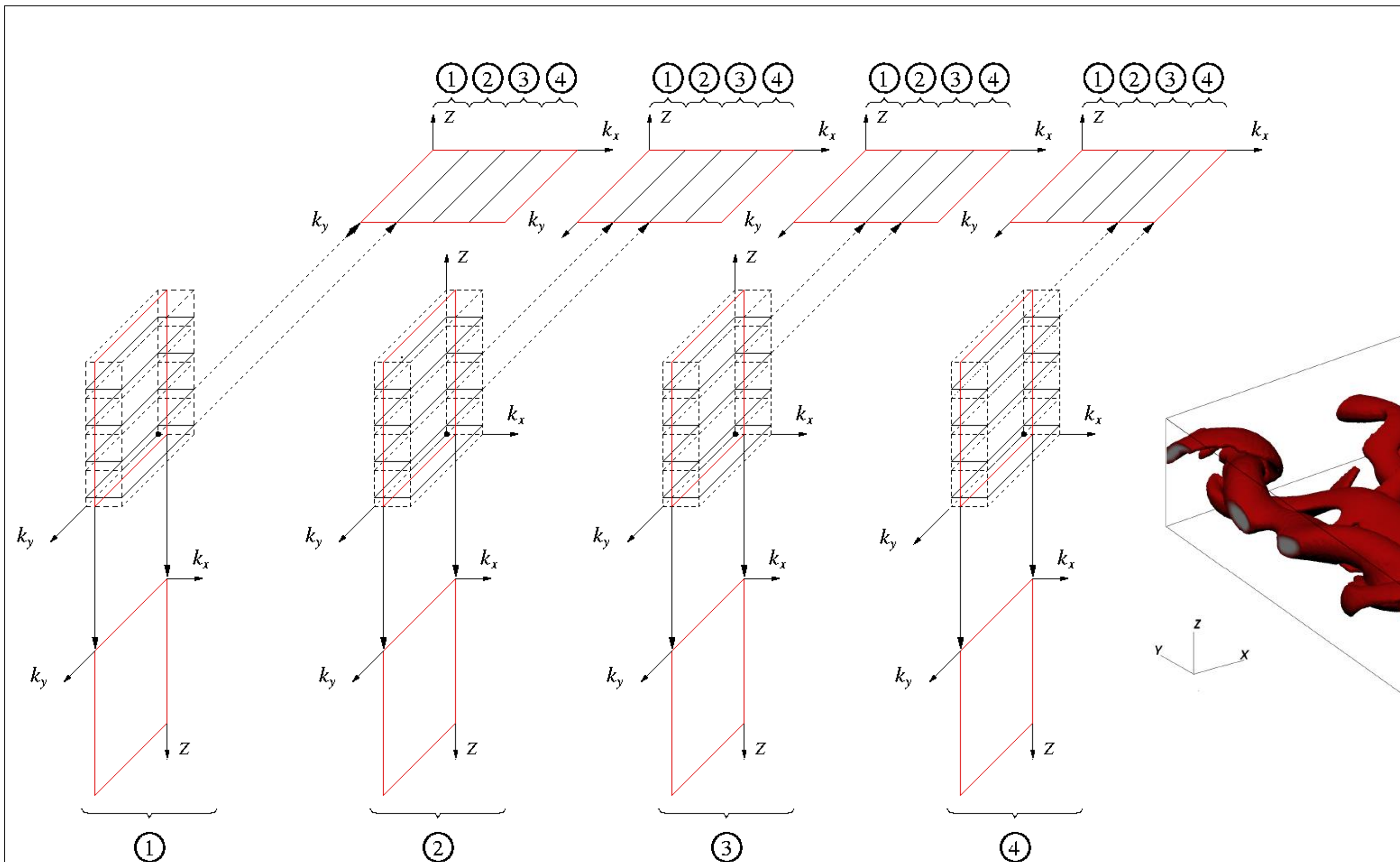
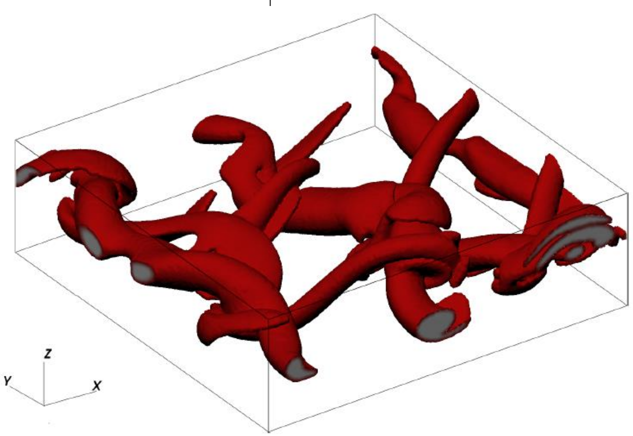
Direct Numerical Simulation (DNS)

The solution of the turbulent flow field is achieved by integration on the NS equations in Spectral space and numerical evaluation of the non linear viscus term by recursive calls to real space





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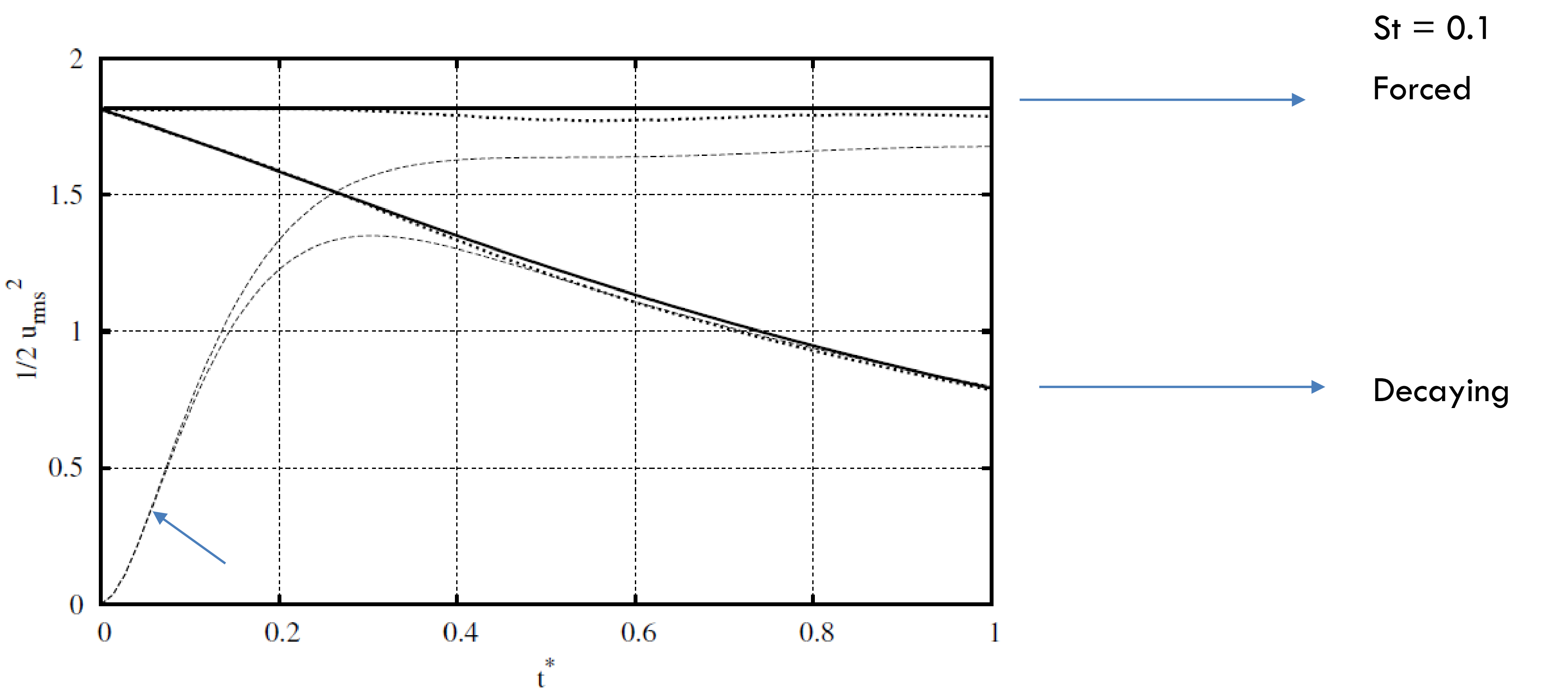
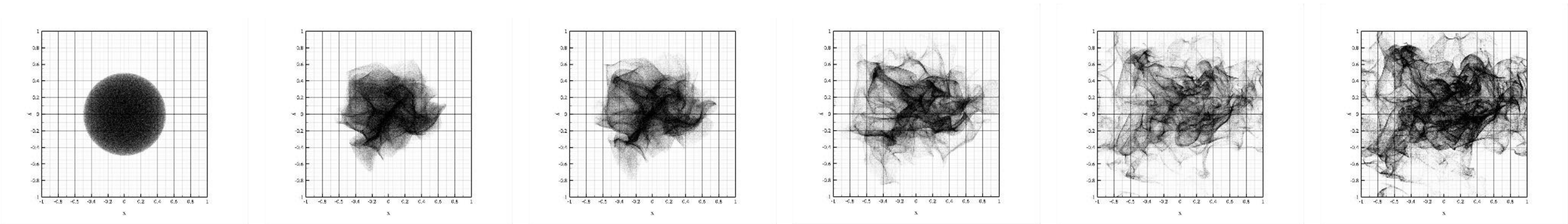
Direct Numerical Simulation Results





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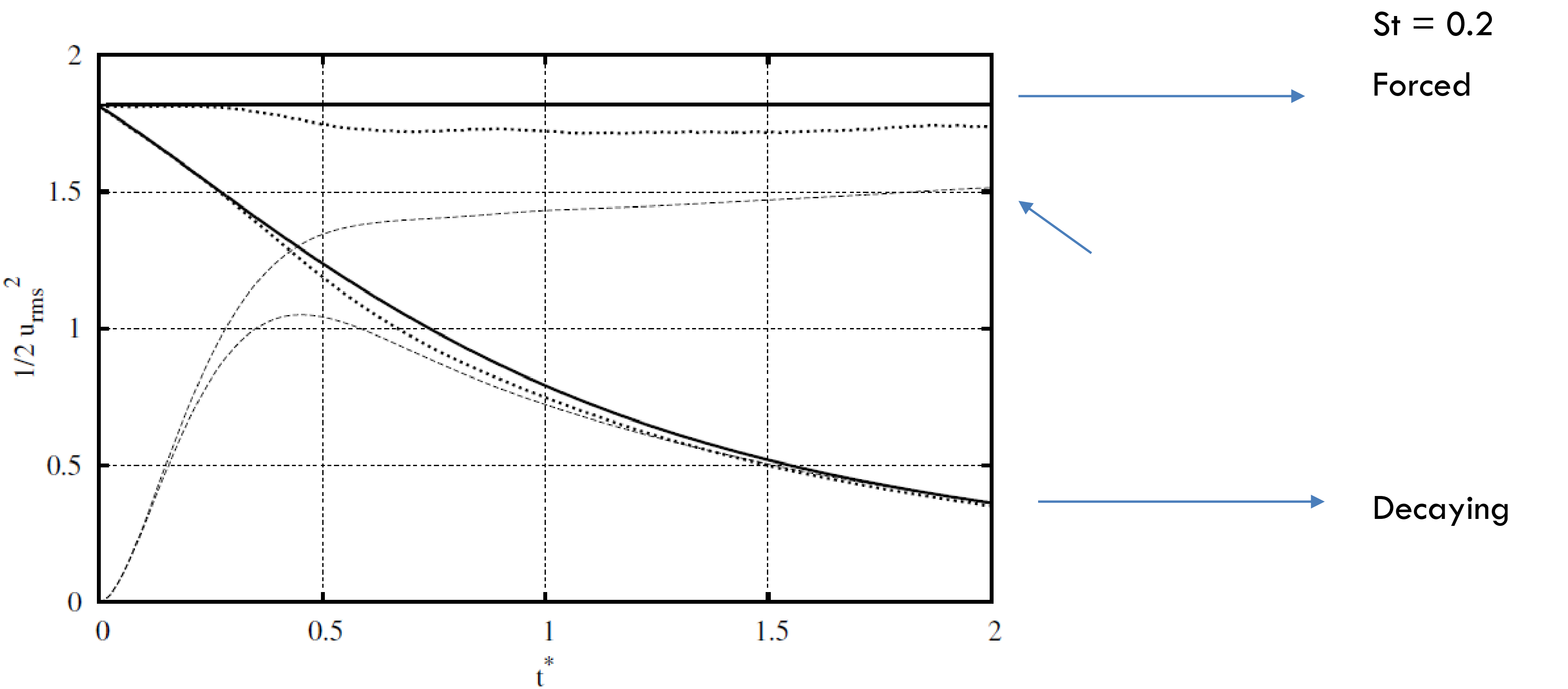

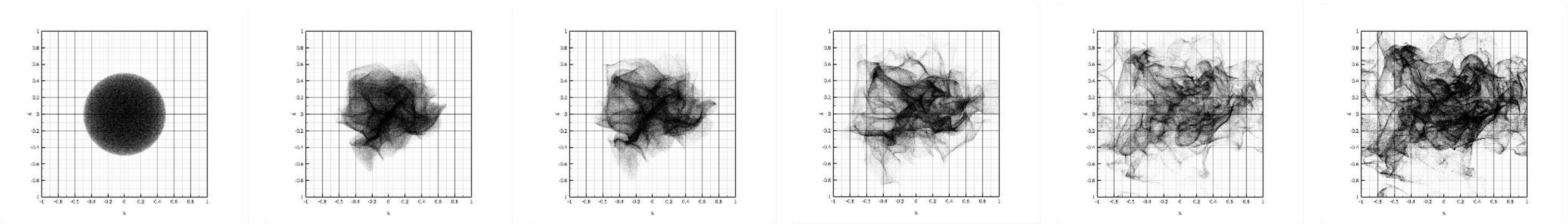

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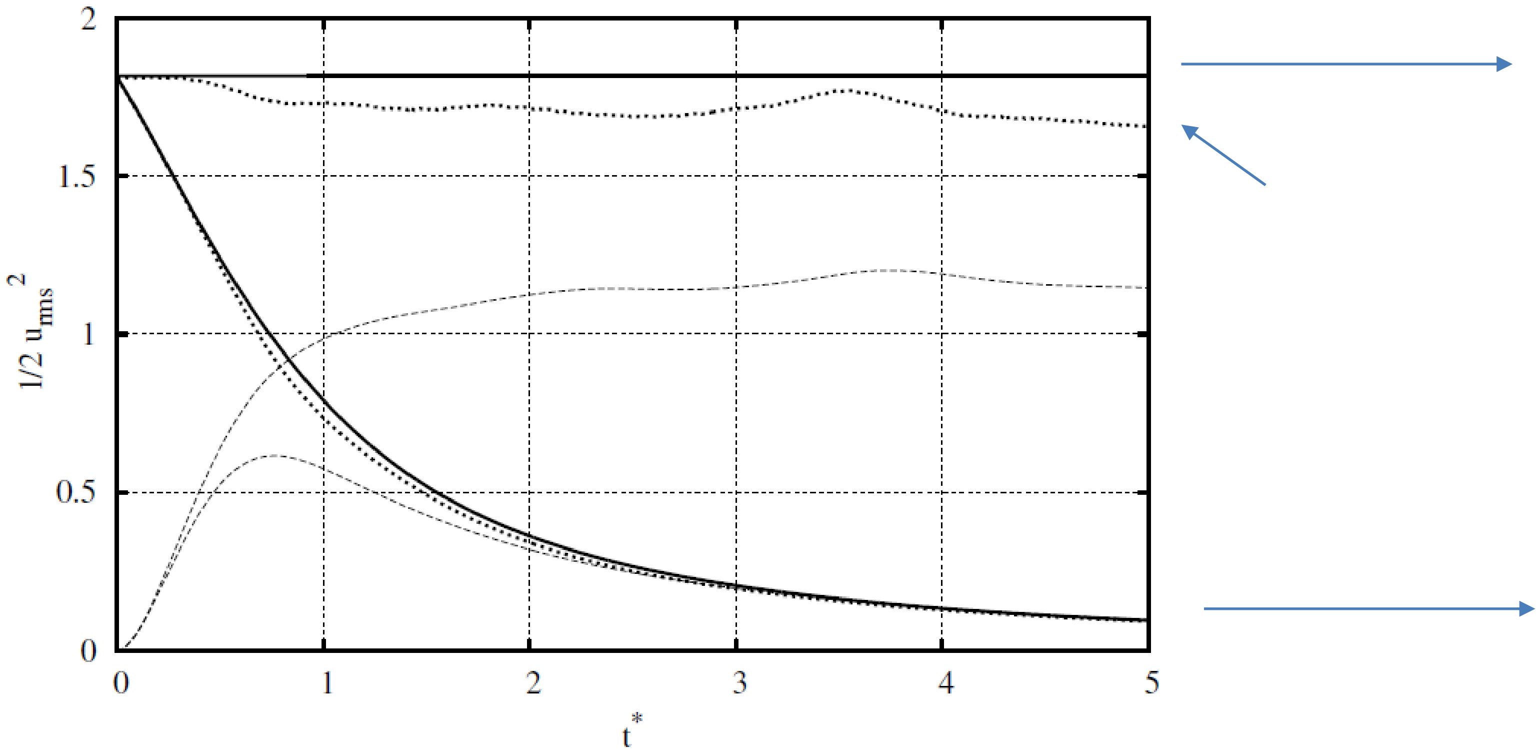
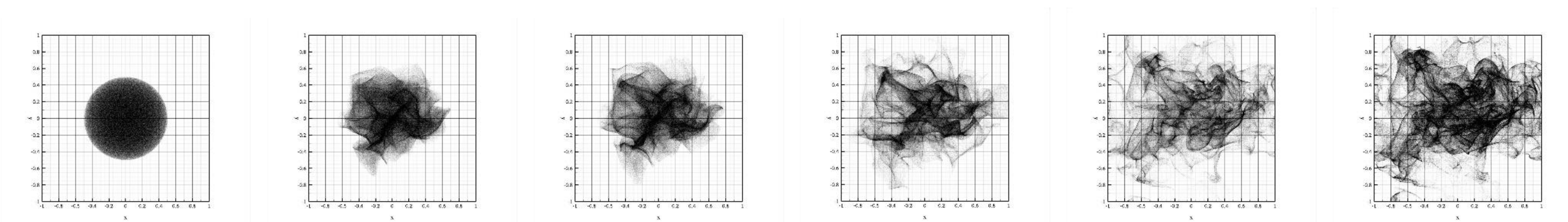
Direct Numerical Simulation Results

St = 0.5

Forced



Decaying



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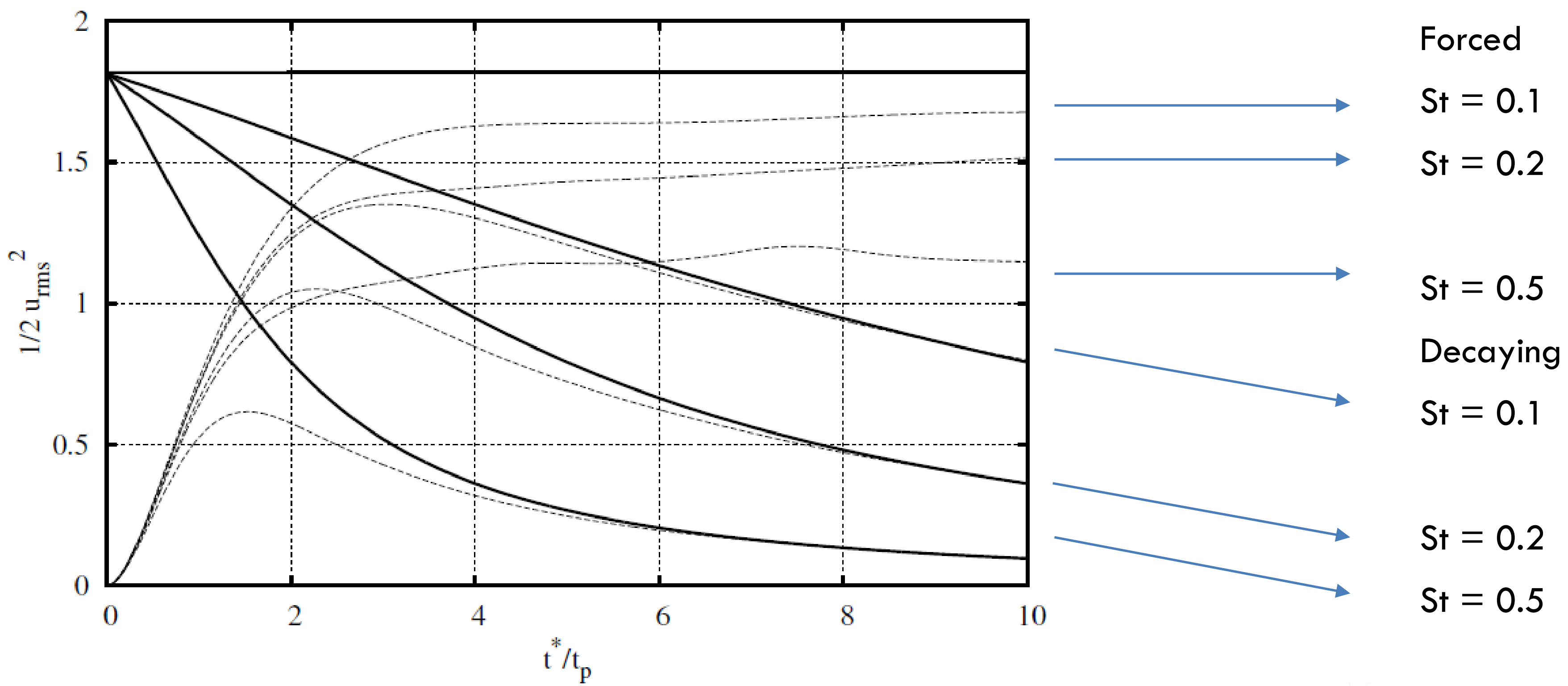
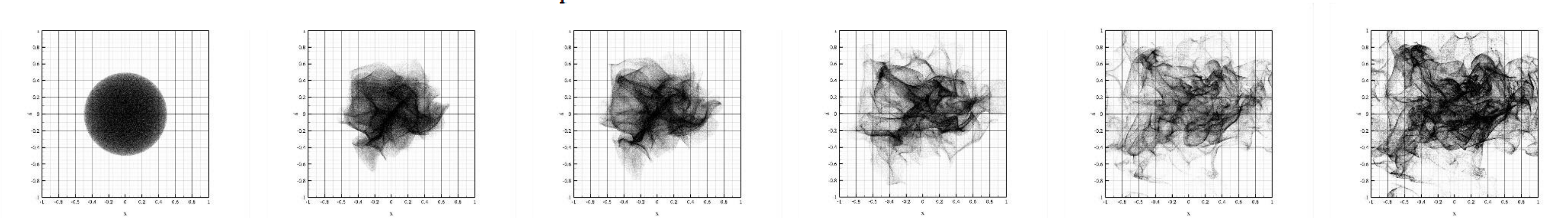
Direct Numerical Simulation Results





Modelling of the evolution of a droplet cloud in a turbulent flow field
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Conclusion

- Particle fluctuations reach a fraction of the turbulent fluctuations depending on St
- The response of the particle fluctuations to turbulence is governed by St
- The dynamic response of the particles to the turbulent fluctuations is significant. The level of adjustment for the particle fluctuations to the turbulent flow field is also a function of the history of the flow fluctuation.

Modelling of the evolution of a droplet cloud in a turbulent flow field
Closure

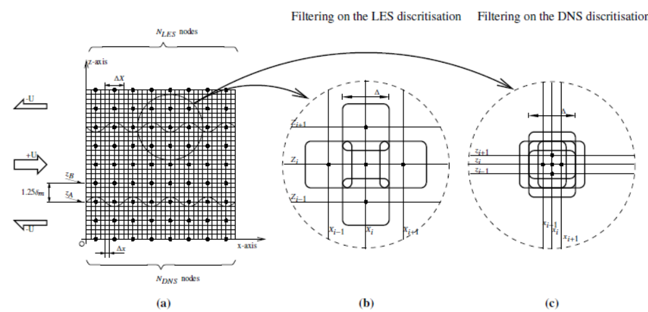
Future Work

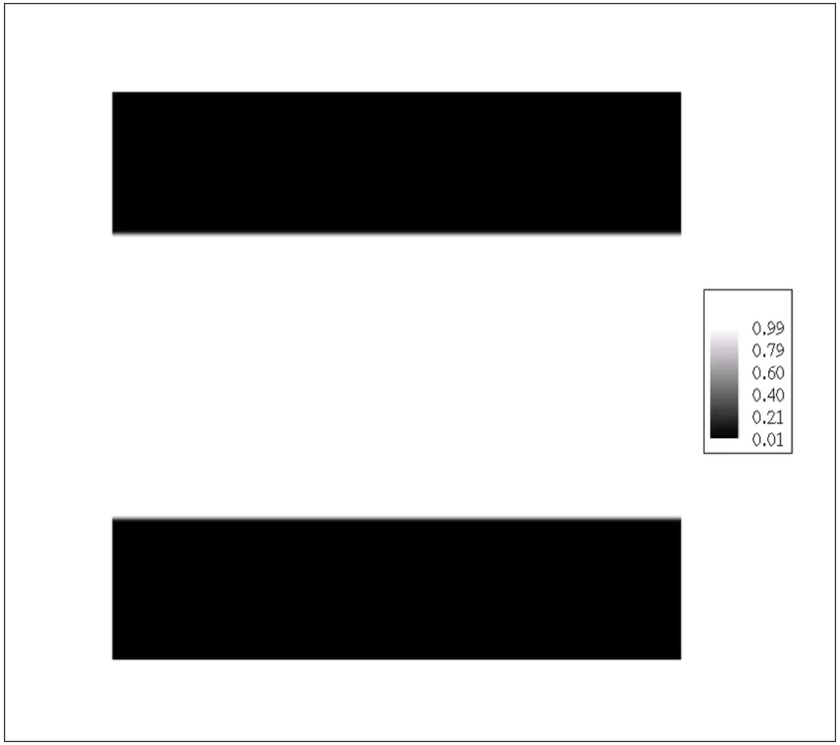
A) Attach the particle fluctuation energy to the particle parcel and account for the history of the particle fluctuations.

B) Impose spatial and temporal filtering operations on DNS databases of fundamental turbulent flows that convey particles aiming to investigate

- The dynamics of the particles fluctuations in non stationary turbulence.
- The turbulent diffusion of particles in real life anisotropic flows (jets) by accounting for the concentration gradients.

C) Implementation of the model suggested in CFD simulations of turbulent flows.





Modelling of the evolution of a droplet cloud in a turbulent flow field

Closure

Thank you for your attention

References

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