Modelling of two-phase vortex ring flow based on the fully Lagrangian Approach

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Outline

• Introduction
• Experimental observations
• FLA. Mathematical formulation
• Results and Discussion:
  – FLA + Analytical Solution
  – FLA + DNS
• Further work
Vortex ring flows

Classical publications:
  Helmholtz 1858
  Lamb 1932
  Saffman 1992
  Shariff & Leonard 1992
  Mohseni & Gharib 1998
+ much more experimental and theoretical studies

Papers from the CAE (SHRL):
  Kaplanski, F., Sazhin, S. S., Begg, S., Fukumoto, Y. & Heikal, M. 2010
  Kaplanski F., Fukumoto, Y. & Rudi, U. 2012
  Danaila, I., Kaplanski, F. & Sazhin, S. 2015
Motivation

EPSRC project “Investigation of vortex ring-like structures in internal combustion engines, taking into account thermal and confinement effects”

Extract from ‘Aims and objectives’: “5. To investigate the applicability of the full Lagrangian approach to modelling sprays in the presence of swirl, thermal gradients, and the heating and evaporation of droplets.”

A typical high-speed photograph of a G-DI spray (Begg et al 2009)
Experimental observations:

Injector 5: Bosch HDEV Hollow Cone, DI piezoelectric, 100-200 bar fuel pressure, high flow rate 42 mg/ms at 200 bar, multiple injection

Vortex ring formation
Why FLA?

Lagrangian frame deformation

Number of folds
Mathematical formulation

One-way coupling

Carrier phase: viscous incompressible liquid
  (DNS and Kaplanski-Rudi solution)

Dispersed phase: identical spherical particles/droplets, pressureless continuum

Force acting on a particle: aerodynamic drag force
Mathematical formulation

Fully Lagrangian approach:

Lagrangian variables:

Coordinates of trajectory origin

+ Time/parameter along a particle trajectory

\[ n_s \left| J \right| = n_s^0 \left| J_0 \right| \]

\[ \frac{dr}{dt} = v_s \]

\[ m \frac{dv_s}{dt} = f_s \]

+ aux. equations for the Jacobian

\[ J_{ij} = \frac{\partial x_i}{\partial x_{0j}} \]
Carrier phase: vortex ring

Incompressible viscous liquid

Cylindrical coordinates

• Kaplanski analytical solution

\[ \Psi = -\frac{r\sqrt{\text{Re}}}{4\sqrt{2t}} \int_0^\infty F \left( x, \sqrt{\text{Re}} \frac{z - z_{vc}}{\sqrt{2t}} \right) J_1 \left( \sqrt{\text{Re}} \frac{x}{\sqrt{2t}} \right) J_1 \left( \sqrt{\text{Re}} \frac{rx}{\sqrt{2t}} \right) dx. \]

• DNS (Second order finite difference)
Dispersed phase equations:

\[ \beta = \frac{6\pi\sigma\mu R_0^2}{m\Gamma_0} \]

\[ n_d r |J| = n_{d0} r_0 \]
\[ \frac{d\mathbf{r}_d}{dt} = \mathbf{v}_d \]
\[ \frac{d\mathbf{v}_d}{dt} = \beta(\mathbf{v} - \mathbf{v}_d) \]
\[ \frac{\partial J_{ij}}{\partial t} = q_{ij} \]
\[ \frac{\partial q_{ij}}{\partial t} = \beta \left( \frac{\partial v_i}{\partial x_1} J_{1j} + \frac{\partial v_i}{\partial x_2} J_{2j} - q_{ij} \right) \]
\[ J_{ij} = \frac{\partial x_{id}}{\partial x_{j0}} \quad q_{ij} = \frac{\partial v_{id}}{\partial x_{j0}} \quad 1 - r \]
\[ r_d = r_{d0}, \quad z_d = z_{d0}, \quad u_d = u_{d0}, \quad v_d = v_{d0}, \quad n_d = n_{d0} \]
\[ q_{ij} = 0, \quad J_{ij} = \delta_{ij} \]

Initial conditions:
Two-phase flow, number density
Simulations based on Kaplanski solution,

$Re = 100$

Two-phase jet

Cloud of particles ahead of the vortex ring
Two-phase flow, number density

Simulations based on DNS

Re = 20 000

Propagation of vortex ring in a cloud of particles
Two-phase jet, number density

Simulations based on DNS

Re = 20 000

Injection

Flow
Further work

• Two-phase jet, injection: more detailed study
• Comparison between DNS+FLA and Kaplanski+FLA
• Evolution of droplet number density in confined vortex rings