Modelling of small droplets heating and evaporation

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Objective

To develop analytical solutions of the equation describing droplet heating in the presence of evaporation, taking into account the moving boundary effect and coupling between liquid and gas phases. To analyse these solutions, and to compare the predictions of these solutions with the results of numerical analysis.

Transient heating of an evaporating droplet

Problem:

$$\frac{\partial T}{\partial t} = \kappa \left(\frac{\partial^2 T}{\partial R^2} + \frac{2}{R} \frac{\partial T}{\partial R} \right) \qquad (1)$$

$$\left(k_l \frac{\partial T}{\partial R} + hT \right) \Bigg|_{R=R_d(t)} = hT_g + \rho_l L \dot{R}_d(t) \qquad (2)$$

Previous models based on const Rd per time step

The solution of pr. (1),(2) was presented as

$$T(R,t) = \frac{R_d}{R} \sum_{n=1}^{\infty} \left\{ q_n e^{-\frac{k\lambda_n^2 t}{R_d^2}} - f_n \mu_0(0) e^{-\frac{k\lambda_n^2 t}{R_d^2}} - f_n \int_0^t \frac{d\mu_0(\tau)}{d\tau} e^{-\frac{k\lambda_n^2 (t-\tau)}{R_d^2}} d\tau \right\} \sin\left(\lambda_n \left(\frac{R}{R_d}\right)\right) + T_{eff}(t),$$

where

$$q_n = \frac{1}{R_d \,||\, v_n \,||^2} \int_0^{R_d} \frac{R}{R_d} T_d(R) \sin\left(\lambda_n \frac{R}{R_d}\right) dR, \quad f_n = \frac{\sin\left(\lambda_n \frac{R}{R_d}\right)}{||\, v_n \,||^2 \lambda_n^2},$$

$$T_{eff} = T_g + \frac{\rho_1 L R_d'}{h}.$$

Alternative presentation of the previous solution

After several transformations

$$T(R,t) = \frac{R_d}{R}\sum_{n=1}^{\infty} \mathrm{Sin}\left[\lambda_n\left(\frac{R}{R_d}\right)\right] q_n\, e^{-\frac{k\lambda_n^2 t}{R_d^2}} +$$

$$T_{eff}\left(1 - \frac{R_d}{R}\sum_{n=1}^{\infty} \mathrm{Sin}\left[\lambda_n\left(\frac{R}{R_d}\right)\right] f_n \frac{k_g}{k_l}\, e^{-\frac{k\lambda_n^2 t}{R_d^2}}\right),$$

where

$$q_n = \frac{1}{R_d\, \|v_n\|^2}\int_0^{R_d} \frac{R}{R_d} T_d(R)\, \sin\left(\lambda_n \frac{R}{R_d}\right) dR, \quad f_n = \frac{\sin\left(\lambda_n \frac{R}{R_d}\right)}{\|v_n\|^2\, \lambda_n^2},$$

$$T_{eff} = T_g + \frac{\rho_l\, L\, R_d{}'}{h}.$$

Alternative presentation of the previous solution

After several transformations

Td(R)

$$T(R,t) = \boxed{\frac{R_d}{R}\sum_{n=1}^{\infty}\text{Sin}\left[\lambda_n\left(\frac{R}{R_d}\right)\right] q_n\, e^{-\frac{k\lambda_n^2 t}{R_d^2}}} +$$

Tg

$$\boxed{T_{eff}\left(1-\frac{R_d}{R}\sum_{n=1}^{\infty}\text{Sin}\left[\lambda_n\left(\frac{R}{R_d}\right)\right] f_n\,\frac{k_g}{k_l}\, e^{-\frac{k\lambda_n^2 t}{R_d^2}}\right)},$$

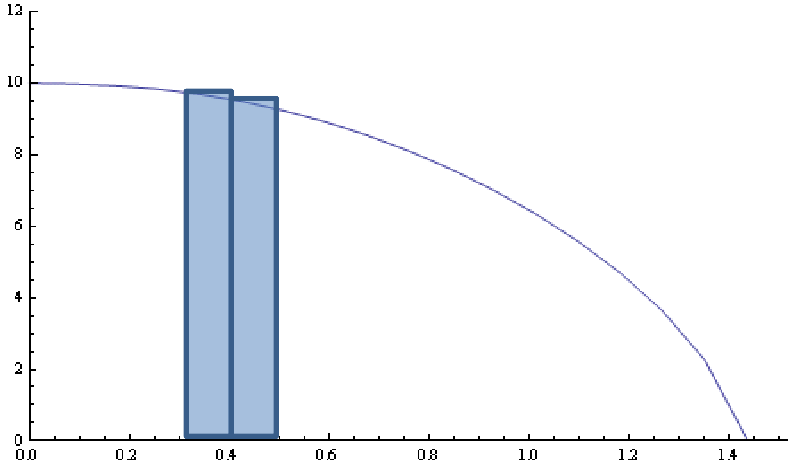
where

$$q_n = \frac{1}{R_d\,||v_n||^2}\int_0^{R_d}\frac{R}{R_d}\,T_d(R)\,\sin\left(\lambda_n\frac{R}{R_d}\right)dR, \quad f_n = \frac{\sin\left(\lambda_n\frac{R}{R_d}\right)}{||v_n||^2\,\lambda_n^2},$$

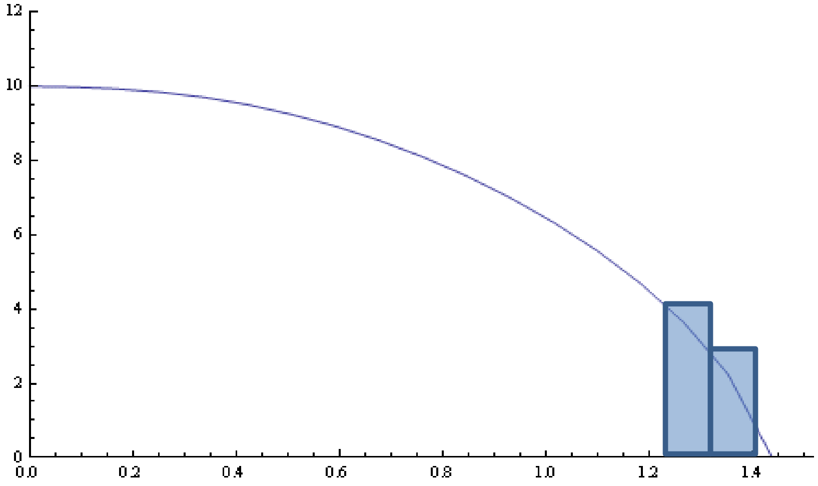
$$T_{eff} = T_g + \frac{\rho_l\, L\, R_d'}{h}.$$

Criticising these models

Everything is alright most of time

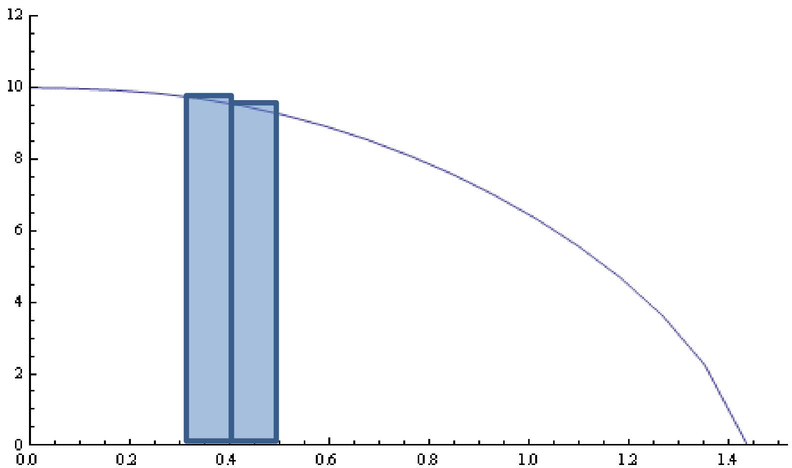


But at the end of evaporation Rd changes too fast, to be approximated as constant



Criticising these models

Everything is alright most of time



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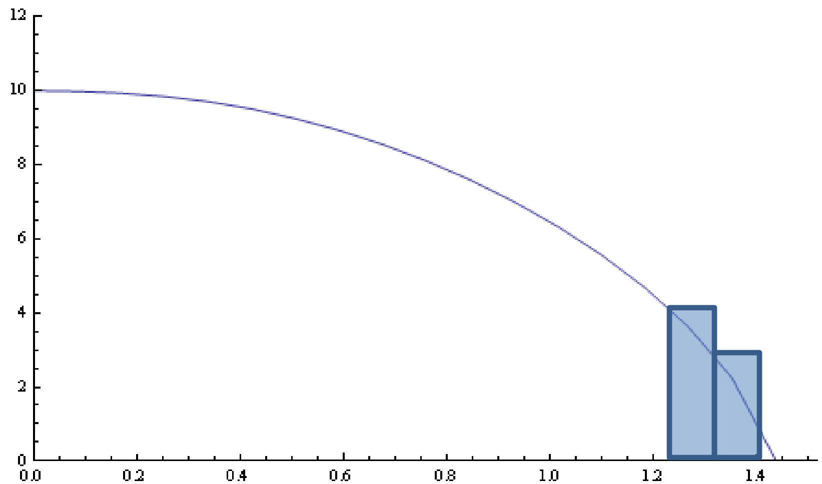


Obvious solution – "decreasing time step" will lead to increasing computational time

Improved model

Let us allow R_d to depend on time:

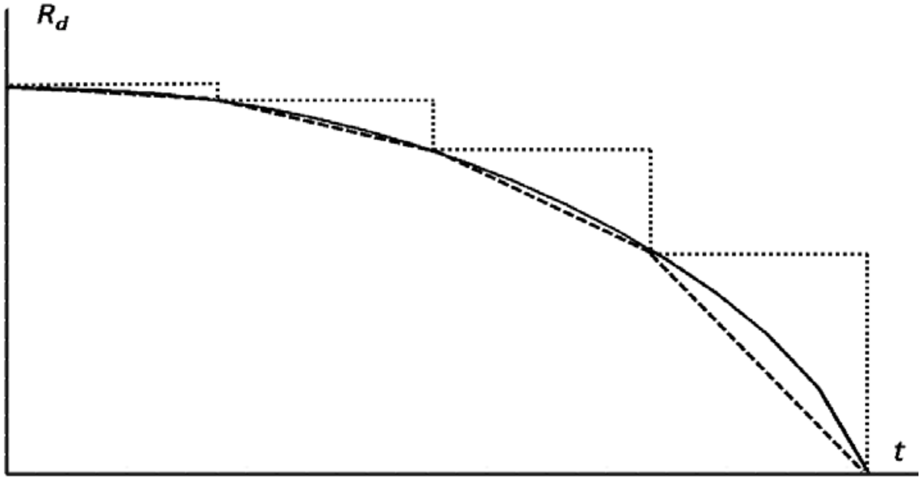
$$R_d(t) = R_{d0}(1 + \alpha t)$$

Improved model

Assuming that

$R_d(t) = R_{d0}(1 + \alpha t)$

The problem at the end of the evaporation time disappeared



Equations are the same

$$\frac{\partial T}{\partial t} = \kappa \left(\frac{\partial^2 T}{\partial R^2} + \frac{2}{R} \frac{\partial T}{\partial R} \right) \qquad (1)$$

$$\left(k_l \frac{\partial T}{\partial R} + hT \right) \Bigg|_{R=R_d(t)} = hT_g + \rho_l L \dot{R}_d(t) \qquad (2)$$

Improved model - Solution

1) $u = T\,R \quad \rightarrow \quad \frac{\partial u}{\partial t} = k\,\frac{\partial^2 u}{\partial R^2}$.

2) $\xi = R \,/\, Rd(t)$.

3) $w = u\,\sqrt{Rd(t)}\;e^{\frac{Rd'(t)\,Rd(t)}{4\,k}\,\xi^2}$.

Finally, we get

$$Rd^2(t)\,\frac{\partial W(t,\xi)}{\partial t} = k\,\frac{\partial^2 W(t,\xi)}{\partial \xi^2} \tag{3}$$

$$\left[\frac{\partial W(t,\xi)}{\partial \xi} + H0(t)\,W(t,\xi)\right]_{\xi=1} = \mu 0(t) \tag{4}$$

Improved model - Solution

$$Rd^2(t)\frac{\partial W(t,\xi)}{\partial t} = k\frac{\partial^2 W(t,\xi)}{\partial \xi^2} \qquad (3)$$

$$\left[\frac{\partial W(t,\xi)}{\partial \xi} + H0(t)\,W(t,\xi)\right]_{\xi=1} = \mu 0(t) \qquad (4)$$

- Assumption that H0 is const
- General case with nonconst H0

Improved model – H0(t) is const

$$V(t,\xi) = W(t,\xi) - \frac{\mu 0(t)}{1+h0}\xi$$

$$Rd^2(t)\frac{\partial V(t,\xi)}{\partial t} = k\frac{\partial^2 V(t,\xi)}{\partial \xi^2} - \frac{\mu 0'(t)}{1+h0}\xi\, Rd^2(t) \qquad (5)$$

$$\left[\frac{\partial V(t,\xi)}{\partial \xi} + h0\, V(t,\xi)\right]_{\xi=1} = 0 \qquad (6)$$

(6) tell us to search solution in form

$$V(t,\xi) = \sum_{n=1}^{\infty} \theta_n(t)\, v_n(\xi)$$

Improved model – H0(t) is const

$$\left[\frac{\partial V(t,\xi)}{\partial \xi} + h0\, V(t,\xi)\right]_{\xi=1} = 0 \qquad (6)$$

$v_n(\xi) = \sin(\lambda_n \xi)$, where

$$\lambda \cos(\lambda_n) + h0 \sin(\lambda_n) = 0$$

$$\theta_n(t) = q_n\, e^{-\frac{k \lambda_n^2 t}{Rd0\, Rd(t)}} + f_n\, \mu 0(t)$$

$$- f_n\, k\, \lambda_n^2 \int_0^t \frac{\mu 0(\tau)}{Rd^2(\tau)}\, e^{\frac{k \lambda_n^2}{\alpha\, Rd0}\left(\frac{1}{Rd(t)} - \frac{1}{Rd(\tau)}\right)}\, d\tau$$

Improved model

The solution is

$$T(R,t) = \frac{1}{\sqrt{R_d(t)}\,R}\, e^{\frac{-R_d'(t)\,R^2}{4\,k\,R_d(t)}} \left(\sum_{n=1}^{\infty} \sin\left(\lambda_n \left(\frac{R}{R_d}\right)\right) q_n\, e^{-\frac{k\,\lambda_n^2\,t}{R_d^2}} - \sum_{n=1}^{\infty} \sin\left(\lambda_n \left(\frac{R}{R_d}\right)\right) f_n\, k\,\lambda_n^2 \int_0^t \frac{\mu_0(\tau)}{R_d^2(\tau)}\, e^{-\frac{k\,\lambda_n^2\,(t-\tau)}{R_d(t)\,R_d(\tau)}}\, d\tau \right),$$

where

$$q_n = \frac{R_{d0}^{3/2}}{\|v_n\|^2} \int_0^1 \xi\, T_d(\xi)\, \sin(\lambda_n\,\xi)\, e^{\frac{R_d'(0)\,R_{d0}^2}{4\,k}\,\xi^2}\, d\xi,$$

$$\mu_0(t) = R_d^{5/2}(t)\, e^{\frac{R_d'(t)\,R_d^2(t)}{4\,k}} \left(\frac{k_g}{k_l}\,\frac{T_g(t)}{R_d(t)} + \frac{\rho_l}{k_l}\, L\, R_d'(t) \right).$$

Improved model

Assuming that $R'_d=0$, the previous expression reduces to the one predicted by the model with R_d = const.

This mean, that the improved model includes the previous one and generalizes it.

Improved model

It could be presented as

$$T(t, R) = \frac{1}{R\sqrt{Rd(t)}} e^{-\frac{Rd'(t)\,R^2}{4\,k\,Rd(t)}}\, W\left(t, \frac{R}{Rd}\right)$$

$$W(t, \xi) = V(t, 1) - \int_0^t \mu 0(\tau)\, G(t, \tau, \xi)\, d\tau$$

where every function is well known.

Improved model – H0(t) is not a const

$$H0(t) = h0 + h1(t)$$

$$h1(t) \neq 0$$

$$\left[\frac{\partial W(t,\xi)}{\partial \xi} + H0(t)\, W(t,\xi)\right]_{\xi=1} = \mu 0(t)$$

$$\left[\frac{\partial W(t,\xi)}{\partial \xi} + h0\, W(t,\xi)\right]_{\xi=1} = \mu 0(t) - h1(t)\, W(t,1)$$

We have the same problem as in previous case

Improved model

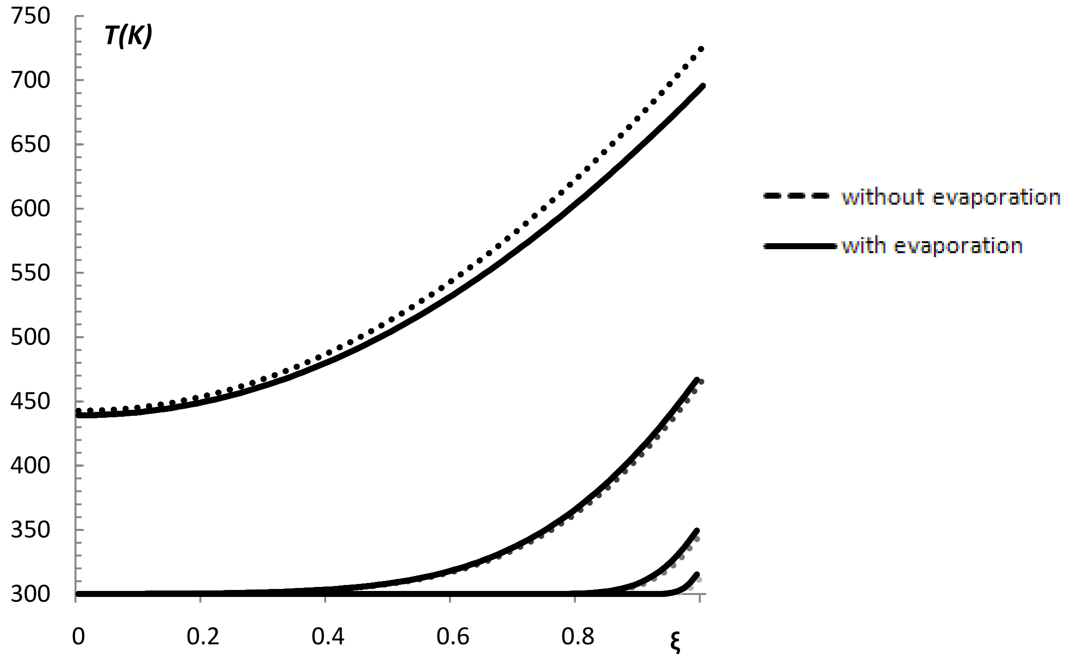
Finally

$$T(t, R) = \frac{1}{R\sqrt{Rd(t)}} e^{-\frac{Rd'(t)\, R^2}{4\, k\, Rd(t)}}\, W\left(t, \frac{R}{Rd}\right)$$

$$W(t, \xi) = V(t, 1) - \int_0^t (\mu 0(\tau) - h1(\tau)\, W(\tau, 1))\, G(t, \tau, \xi)\, d\tau \quad (9)$$

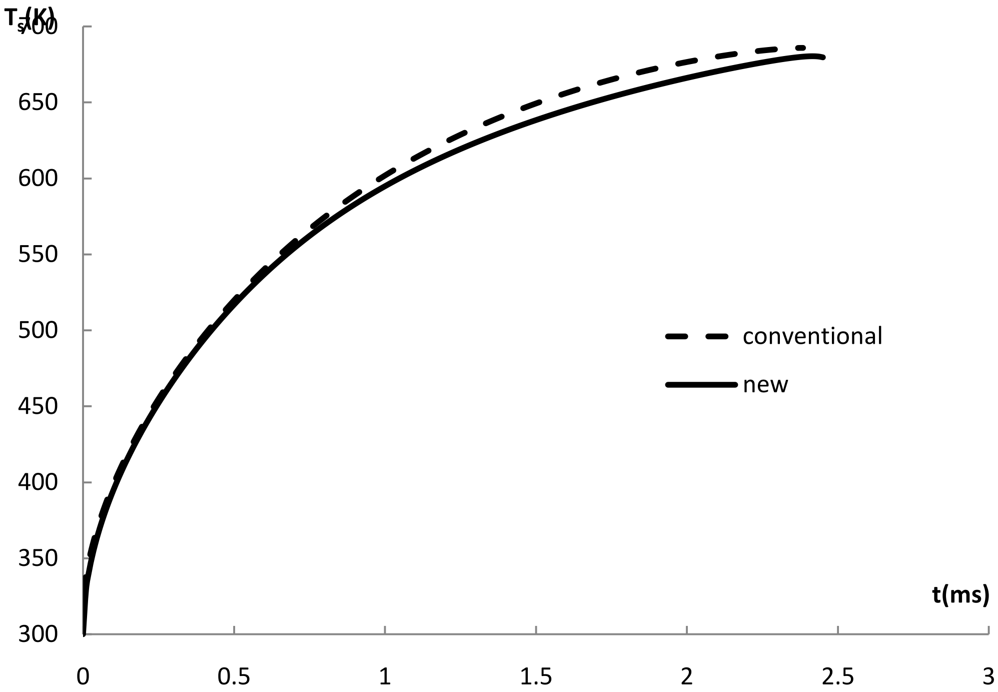
where every function is well known and (9) is Volterra integral equation.

Results



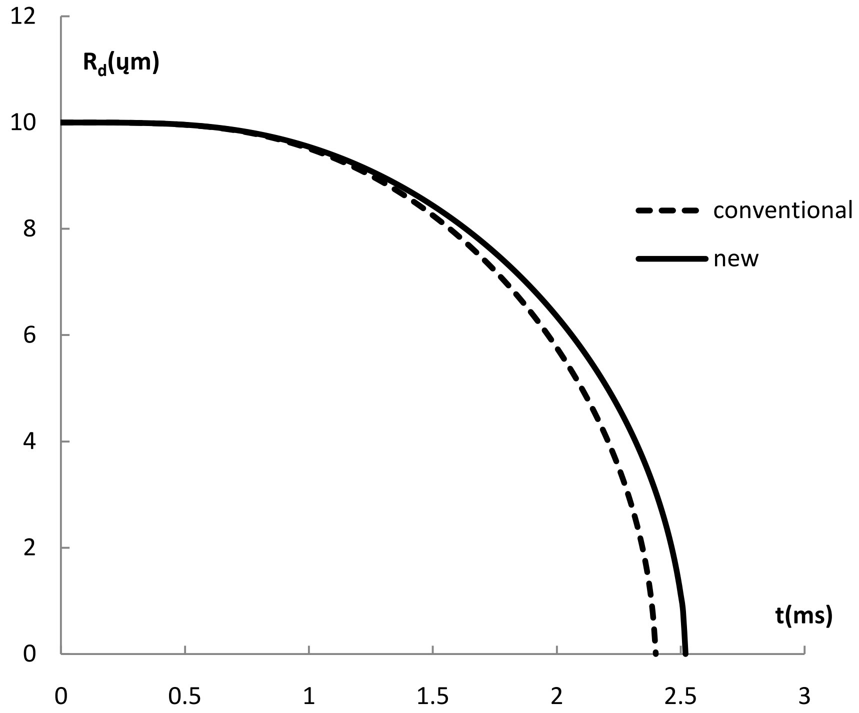
T(K)
750
700
650
600
550
500
450
400
350
300
0
0.2
0.4
0.6
0.8
ξ
without evaporation
with evaporation

Results



T_s(K)
700
650
600
550
500
450
400
350
300
0
0.5
1
1.5
2
2.5
3
t(ms)
conventional
new

Results



Next steps

- Results with another models and experimental data
- Take into account droplet movement, thermal dilatation, radiation and coupling between liquid and gas phases
- Implement solution into a CFD code

Thank you

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