A simple model investigating the break-up length of jets

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Outline

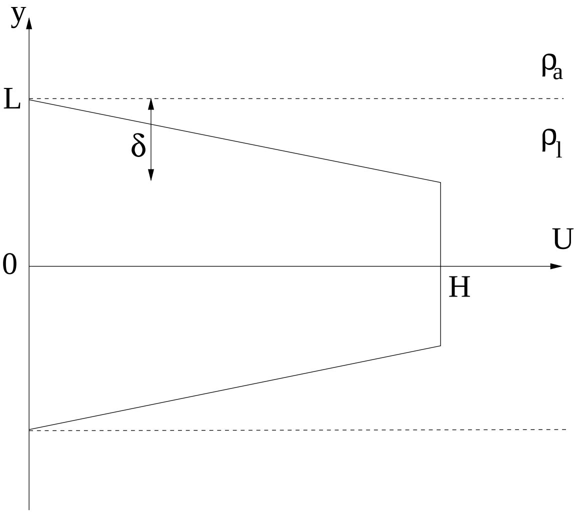
Research objectives

Understand how acceleration affects the stability and subsequent break-up of jets.

Acceleration is observed to increase break-up length in Diesel jets. Is this true of other jets?

For now we are not interested in quantitative agreement with experiments, we are trying to understand the mechanisms behind observations. We do this by a simple mathematical model.

A simple jet model



This model is not the best, but was the first considered.

Short bit of Mathematics

Linear stability characteristics governed by Rayleigh's equation

$$(\alpha U - \omega)\left(\frac{d^2 v}{dy^2} - \alpha^2 v\right) - \alpha U'' v = 0.$$

By solving this in each fluid layer and matching across the interface we derive a dispersion relation which relates the wavenumber α and the frequency ω of the disturbance.

$$D(\alpha, \omega) = c_2 \omega^2 + c_1 \omega + c_0 = 0.$$

Methods of solution include: Choosing α real and solving for ω complex (Temporal Stability - Stability of jet in time). Choosing ω real and solving for α complex (Spatial stability - Stability of jet in space).

We assume that all disturbance frequencies are generated in jet at equal amplitudes (reasonable assumption).

As disturbance waves are dispersive we decide to study stability of wave packets. i.e. growth of disturbance packets along characteristics $x/t = \mathrm{real\ constant}$. This method assumes both α and ω are complex and we solve for a complex growth rate.

$$g = \omega - (x/t)\alpha$$

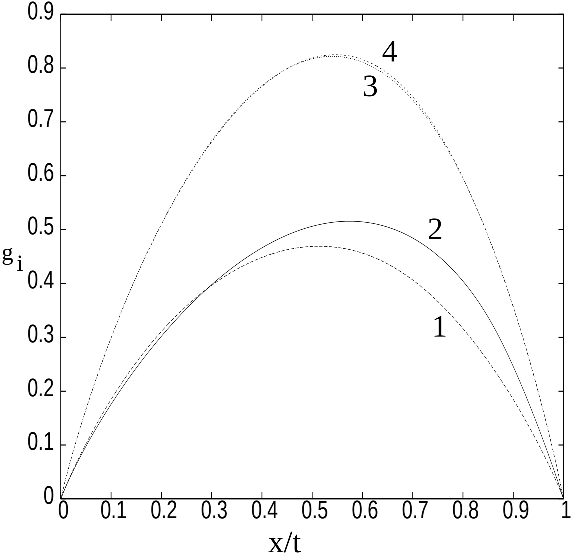
For transient stability we cannot just consider the magnitude of the transverse perturbation $v(x, y, t)$. For steady flows considering the relative size of this perturbation is sufficient.

However if $|v| = |v_0|$ in velocity profile U and $|v| = 2|v_0|$ in the velocity field $2U$ then these systems are as unstable as one another. Thus the important ratio to consider is $\max|v/U|$.

The we can define the break-up length via

$$x_b = \frac{\partial \omega}{\partial \alpha} g_i^{-1} \left(\ln \left| \frac{v_b}{H} \right|_{\max} - \ln \left| \frac{v_0}{H} \right|_{\max} \right)$$

Typical growth rate curves. $q = \rho_l/\rho_a = 1/500$ (typical for diesel-air)

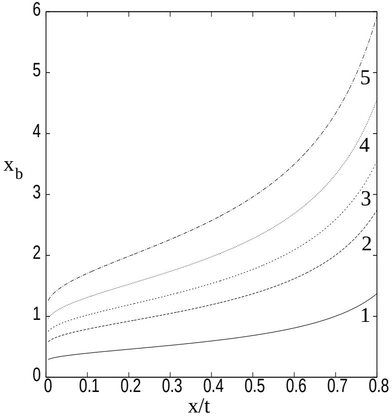


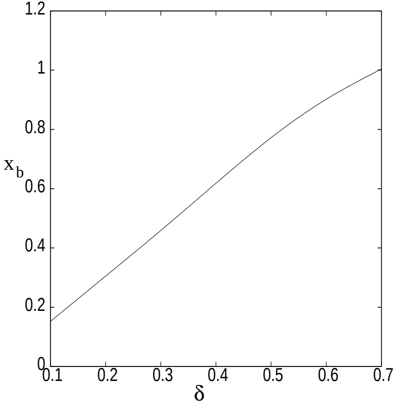
We are interested in when jet first breaks up.

Key assumptions of accelerating jet profile

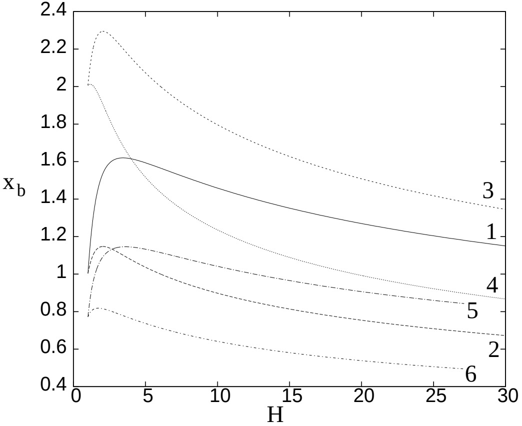
- When a jet accelerates the shear layer in the liquid thins. Obvious from nozzel flow.
- $|v_b/H|$ remains fixed, but $|v_0/H|$ changes with H. How does this change?
- How exactly does the velocity profile change with time in a 'real' simulation?

Two mechanisms at work.





If both mechanisms combine



Outline

Questions:

What is the significance of the air layer?

How does the velocity profile actually behave for diesel jets?

What frequencies are actually present in the diesel jet process?

How do other jets behave? Is surface tension significant?